Gray solitary-wave solutions in nonlinear negative-index materials

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We predict the existence of gray (dark) solitary waves in negative-index materials on the basis of a derived higher-order nonlinear Schrödinger equation. The conditions for the formation of three cases of gray solitary waves and exact analytical expressions are presented. Furthermore, we investigate the properties of these gray solitary waves in negative-index materials. The results show that the higher-order linear and nonlinear effects play a crucial role for the formation and properties of the second and third cases of gray solitary waves.

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I. INTRODUCTION

Since the experimental confirmation of Veselago’s prediction of negative-index materials (NIMs) with simultaneously negative electric and magnetic responses [1,2], NIMs have attracted a lot of interest due to the remarkable property of having an antiparallel phase velocity and Poynting vector [3–9]. Recently, nonlinear effects in NIMs have been extensively studied, including second-harmonic generation [10], parametric amplification [11], modulation instability [12], and soliton propagation [13–21]. It is well known that nonlinear NIMs not only possess dispersive magnetic permeability, which leads to the difference between the dynamic models for the envelopes of the electromagnetic wave in NIMs and in ordinary materials, but also support rich localized modes [13–21]. These localized modes in nonlinear NIMs can take the forms of gap solitons [13], spatial solitons [14], spatiotemporal solitons [15,16], and temporal solitons [17–21]. In particular, small-amplitude dark and bright solitons on the background of a continuous wave in NIMs have been obtained by employing small-amplitude soliton approximation method [20]. Dark solitons and their interactions in metamaterials have been studied by using a Korteweg-de Vries description [21]. To best of our knowledge, the study on gray (dark) solitons in NIMs still remains new.

The properties of dark solitons in ordinary materials have been well investigated both in theories and experiments [22–28]. Dark solitons are generally considered to be less desirable for applications in high-speed communication systems because of their higher average power, but they have better stability against material loss and background noise than bright solitons do [24]. Moreover, the time jitter in dark solitons is much lower, and the interaction between neighboring dark solitons is much less than that for their bright counterparts [25–27]. Since there are striking differences between ordinary materials and NIMs, in which the former have a constant permeability and positive refraction while the latter have a dispersive permeability and negative refraction, it is necessary to investigate the properties of dark solitons in NIMs. In this paper, motivated by the work of Scalora et al. [18], we derive an extended nonlinear Schrödinger (NLS) equation including third-order dispersion (TOD) and second-order nonlinear dispersion effects, which describes the propagation of few-cycle electromagnetic pulses in nonlinear NIMs. Furthermore, we present three cases of exact gray (dark) solitary-wave solutions for this equation by ansatz method and investigate the formation conditions and properties of these solutions in detail.

II. THEORETICAL MODELS

NIMs are composed of a regular array of unit cells whose size is usually much smaller than the wavelengths of propagating electromagnetic waves. Therefore, NIMs may be considered as continuous and homogeneous according to effective-medium theory and may be described by effective \( \varepsilon(\omega) \) and \( \mu(\omega) \) [17,18]. Let us consider the propagation of electromagnetic wave in an isotropic and homogeneous NIM, whose \( \varepsilon(\omega) \) and \( \mu(\omega) \) can be expanded in Taylor series [18].

\[
\omega \varepsilon(\omega) = \sum_{n=0}^{\infty} \left( \varepsilon^{(n)}(\omega) \right) \frac{(\omega - \omega_0)^n}{n!}, \quad (1a)
\]

\[
\omega \mu(\omega) = \sum_{n=0}^{\infty} \left( \mu^{(n)}(\omega) \right) \frac{(\omega - \omega_0)^n}{n!}, \quad (1b)
\]

where \( \omega_0 \) is the carrier frequency of the incident electromagnetic wave. Substituting Eqs. (1) into Maxwell’s equations, one can obtain a system of coupled nonlinear Schrödinger equations, [18]}

\[
\frac{\partial E(z,t)}{\partial z} = i e^{ik_z z} \sum_{n=0}^{\infty} \left( \frac{\varepsilon^{(n)}}{n!} \frac{\partial^{n} \varepsilon(z,t)}{\partial t^{n}} \right), \quad (2a)
\]
\[
\frac{\partial H(z,t)}{\partial z} = -i e^{i(\omega t - \omega z)} \sum_{n=0}^{\infty} \left( i \frac{\partial^{n} \omega_{\omega}}{\partial \omega^{n}} \right) \left( 1 \frac{\partial E(z,t)}{\partial t^{n}} \right) - \frac{1}{c} \frac{\partial}{\partial t} P_{\mu}(z,t),
\]

(2b)

where \( E(z,t) = \Omega(z,t) e^{i(\omega_{\omega} - \omega)} \) and \( H(z,t) = \Omega(z,t) e^{i(\omega_{\omega} - \omega)} \).

Here, \( E(z,t) \) and \( H(z,t) \) are the envelopes of the electric and magnetic fields, respectively. \( P_{\mu} = \chi^{(3)}|E|^{2}E + \chi^{(5)}|^E|^{4}E + \ldots \) is the nonlinear polarization and \( \gamma_{0} \) is the non-linear susceptibility. It should be noticed that, in typical nonlinear NIMs, there exists significant magnetic nonlinearity \([5] \). However, the symmetry of Eqs. (2a) and (2b) for electric field \( E \) and magnetic field \( H \) suggests that nonlinear magnetization produces qualitatively similar effects \([18] \), leading to similar propagation equation for the magnetic field. Therefore, here we consider the wave propagation in the case of nonlinear polarization. Equations (2) are very general because they include any desired order of dispersion and susceptibility. In order to investigate the higher-order linear and nonlinear effects on the propagation of ultrashort electromagnetic waves, here we retain linear derivatives up to the third-order and neglect nonlinear third-order temporal derivatives; thus, Eq. (2a) can be simplified as

\[
\alpha \frac{\partial E}{\partial t} + \frac{\alpha^{\prime}}{4 \pi} \frac{\partial E}{\partial \tau} + \frac{1}{64 \pi^{3}} \frac{\partial E}{\partial \tau^{3}} = \beta \omega_{\mu} - i \beta n H - \frac{\partial H}{\partial \zeta} + i \beta \chi^{(3)}
\]

\[
\times |E|^{2}E - \chi^{(3)} \frac{\partial}{\partial \tau}(|E|^{2}E),
\]

(3a)

where \( \alpha = \frac{\partial \omega_{\omega}}{\partial \omega_{\mu}}, \alpha^{\prime} = \frac{\partial^{2} \omega_{\omega}}{\partial \omega_{\omega}^{2}}, \gamma = \frac{\partial \omega_{\mu}}{\partial \omega_{\mu}}, \gamma^{\prime} = \frac{\partial^{2} \omega_{\mu}}{\partial \omega_{\mu}^{2}} \).

\[
\frac{\partial H}{\partial \tau} + \frac{\gamma^{\prime}}{4 \pi} \frac{\partial H}{\partial \tau} + \frac{1}{64 \pi^{3}} \frac{\partial H}{\partial \tau^{3}} = \beta \mu_{\Omega} - i \beta n H - \frac{\partial E}{\partial \zeta},
\]

(3b)

Differentiating Eq. (5) with respect to \( \zeta \) and \( \tau \), respectively, and neglecting the fourth-order linear derivative and third-order nonlinear temporal derivative \([30] \), we can estimate \( \frac{\partial E}{\partial \zeta} \) and \( \frac{\partial E}{\partial \tau} \) in the following approximations:

\[
\frac{\partial E}{\partial \zeta} = \frac{i \beta \mu_{\Omega}^{(3)}}{2n} \frac{\partial}{\partial \zeta} |E|^{2}E - \frac{\beta \mu_{\Omega}^{(3)} k_{0}^{2}}{4n} \frac{\partial^{2}}{\partial \tau^{2}} |E|^{2}E,
\]

(6a)

\[
\frac{\partial E}{\partial \tau} = \frac{i \beta \mu_{\Omega}^{(3)}}{2n} \frac{\partial}{\partial \tau} |E|^{2}E + \frac{\beta \mu_{\Omega}^{(3)} k_{0}^{2}}{2n} \frac{\partial^{2}}{\partial \tau^{2}} |E|^{2}E + \frac{i k_{2}^{2} \partial^{2}}{2 \partial \tau^{2}},
\]

(6b)

where \( k_{0} = (1/\beta n) \left[ (k_{2} / V_{g}) (1 / V_{g}) \right] \) is the group velocity dispersion (GVD) coefficient. Substituting Eqs. (6) into Eq. (5), we can obtain a generalized higher-order NLS equation:

\[
\frac{\partial E}{\partial \zeta} = \frac{ik_{2}^{2} \partial^{2} E}{2 \partial \tau^{2}} + \frac{k_{2}^{2} \partial^{2} E}{2 \partial \tau^{2}} + ip_{3} |E|^{2}E - ip_{3} |E|^{2}E + \frac{\partial}{\partial \tau} |E|^{2}E,
\]

(7)

From Eq. (4), one can see that the wave propagates at a group velocity \( v_{g} = 2n/(\epsilon \gamma + \mu \alpha) \) (in units of \( c \)). For relatively transparent NIMs \([18,29] \), \( \alpha > 0, \gamma > 0, \) and \( n < 0; \)

therefore, \( v_{g} \) is always positive. In addition, it should be noticed that Eq. (4) is not suitable for the case of \( n \rightarrow 0 \). In this case one should solve Eqs. (3) directly. Introducing a retarded coordinate \( \partial \zeta = \partial \zeta + (1/V_{g})(\partial \zeta/\partial \tau) \), Eq. (4) can be written as

\[
\frac{\partial E}{\partial \tau} = \frac{i}{2 \beta n} \left( \frac{\partial^{2} E}{\partial \tau^{2}} - \alpha \frac{\partial E}{\partial \tau} \right) + \frac{1}{8 \pi \beta n} \left( \alpha^{\prime} \gamma + \gamma^{\prime} \alpha + \beta \frac{\partial E}{\partial \zeta} \right) \frac{\partial}{\partial \tau} |E|^{2}E + \frac{i \beta \mu_{\Omega}^{(3)}}{2n} \frac{\partial}{\partial \tau} |E|^{2}E + \frac{\beta \mu_{\Omega}^{(3)} k_{0}^{2}}{2n} \frac{\partial^{2}}{\partial \tau^{2}} |E|^{2}E + \frac{i k_{2}^{2} \partial^{2}}{2 \partial \tau^{2}} |E|^{2}E.
\]

(5)
are not isolated, but comply with the relation presented in Fig. 1 at a given plasma frequency. In the following, we will investigate the formations and properties of gray solitary waves and take values of the model parameters based on the relation in Fig. 1.

III. EXACT GRAY SOLITARY-WAVE SOLUTIONS

To search for gray solitary-wave solutions of Eq. (7), we take an ansatz as follows:

$$E(\xi, \tau) = (\lambda + i\rho \tan[\eta(\tau - \chi)]) \exp[i(k\xi - \Omega \tau)],$$

whence intensity is \( |E(\xi, \tau)|^2 = \lambda^2 + \rho^2 \tan^2[\eta(\tau - \chi)] \). It is clear to see that Eq. (8) denotes a gray (dark) solitary wave when \( \lambda = 0 \) (\( \alpha = 0 \)), where \( \lambda \) and \( \rho \) are, respectively, related to the amplitude and the dip depth of the gray solitary wave, and \( \eta, \chi, k, \) and \( \Omega \) are the respective real parameters describing inverse pulse width, inverse group velocity, wave number, and frequency shift. Substituting Eq. (8) into Eq. (7) and setting the coefficients of independent terms equal to zero, we can obtain six compatible equations:

\[
\begin{align*}
2(k\lambda - \eta\rho\chi) &- 2k_3(2\eta\rho - k_2\Omega) \lambda - \lambda^3(p_3 - p_5\lambda^2) \\
- 2\lambda^2s_1(2\eta\rho - 2\Omega\lambda) + 2k_3(2\eta\rho + 3\eta\rho\Omega^2 - \lambda\Omega^2) \\
&+ 2\lambda s_2(2\rho^2\eta^2 + 2\eta\Omega\lambda - \lambda^2\Omega^2) = 0, \\
&\text{(9a)} \\
2k &+ 2k_3(2\eta\rho + \lambda\Omega) + 2k_3(2\eta\rho^2 + \Omega^2) - 2\lambda^2(p_3 - p_5\lambda^2) \\
&- 2k_3\Omega(6\eta^2 + \Omega^2) \\
- 2s_2(2\eta\lambda - 6\eta^2\rho^2 + 4\eta\rho\lambda\Omega + \lambda^2\Omega^2) = 0, \\
&\text{(9b)} \\
2(k_1\chi + k_2\Omega) &- 8k_3\eta_2 - 3k_3\Omega^2 + s_1(\eta_2^2 - 3\eta^2\rho^2 + \lambda\rho\Omega) \\
&- \lambda\rho(p_3 + 2p_5\lambda^2) \\
- s_2(8\eta\rho\lambda + 2\eta\lambda^2\Omega - 6\eta^2\rho^2\Omega + \lambda\rho\Omega^2) = 0, \\
&\text{(9c)} \\
\eta_2(2k_2 - 6k_3\Omega - 2k_5\eta^2 - 18\eta^2s_2) &- \rho\Omega s_3(4\eta\lambda - \rho\Omega) \\
+ \rho(p_3 - 2\rho^2p_5 + 2\eta\lambda s_1 - \rho\Omega s_3) = 0, \\
&\text{(9d)} \\
6k_3\eta_3 &+ p_5\lambda\rho^3 + 3\eta\rho^2s_1 + 6\eta\rho(\eta\lambda - \rho\Omega)s_2 = 0, \\
&\text{(9e)} \\
p_5s_3 &+ 2\eta^2s_2 = 0. \\
&\text{(9f)}
\end{align*}
\]

Solving Eqs. (9a)–(9f), we find that there exist three cases of exact gray (dark) solitary-wave solutions for Eq. (7).

(i) When all higher-order effects are neglected, i.e., \( k_3 = p_4 = s_1 = s_2 = 0 \), Eq. (7) is reduced to a standard NLS equation which admits the following gray solitary-wave solution:

\[
E(\xi, \tau) = \left(\lambda + i\eta \sqrt{-\frac{k_2}{p_3}} \right) \tanh\left[\eta \left(\tau + \left(\frac{p_4}{k_2} + k_2\Omega \right) \xi\right)\right] \exp\left[i\left(\lambda^2p_3 - \eta^2k_2 - \frac{k_3\Omega^2}{2}\right)\xi - \Omega \tau\right].
\]

\[
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\]

FIG. 1. (Color online) Curves of \( k_2, k_3, p_3, p_5, s_1, \) and \( s_2 \) versus \( \bar{\omega} \) in focusing NIMs for \( \omega_m/\omega_p = 0.8 \). Here, \( p_3, s_1, \) and \( s_2 \) are plotted in units of \( \chi^{(3)} \), and \( p_5 \) in units of \( (\chi^{(3)})^2 \). Inset: curves of \( e, \mu, \) and \( n \) versus \( \bar{\omega} \) for \( \omega_m/\omega_p = 0.8 \).
plies that the gray solitary wave can exist in abnormal
depths related to
This is different from that in ordinary materials. Moreover,
solitary wave nonlinearity must satisfy the condition
which was obtained in nonlinear magnetic metamateri-
als. As shown in the inset, our analytical results are in good
agreement with the numerical simulations.

\begin{equation}
\chi = -\frac{k_2^2}{16k_3} + \frac{3k_3p_3^2}{4s_1^2} - \frac{k_3p_3}{4s_1} - \left(\frac{3\lambda^2}{2} + \rho^2\right)s_1, \quad (11c)
\end{equation}

where $\lambda$ is a free parameter. Equation (11a) implies that the TOD $k_3$ and self-steepening $s_1$ must satisfy $s_1k_3 < 0$, and the width $\eta$ and the dip depth $\rho$ are related to each other, which just reflects the intrinsic characteristics of solitons. Other soliton parameters $\chi$, $k$, $\eta$, and $\Omega$ are also related to the TOD $k_3$ and the self-steepening $s_1$ [see Eqs. (11b)–(11d)]. Figures 3(a) and 3(b) present the dependences of the velocity $\chi$, wave number $k$, width $\eta$, and frequency shift $\Omega$ on the TOD $k_3$ and the self-steepening $s_1$, respectively. It is clear that the influence of $k_3$ and $s_1$ on $\chi$ and $k$ is much stronger than on $\Omega$ and $\eta$. This is the reason why in Fig. 3(c), the width of the gray solitary wave (11) hardly changes, while its velocity is distinctly different under different $k_3$'s and $s_1$'s. These results show that the TOD and the self-steepening effects not only influence the existence condition of this gray solitary wave (11), but also dominate its propagation properties in NIMs. Figure 3(d) shows the evolution of this gray solitary wave (11) in NIMs for $\omega_0/\omega_p=0.9$ and $\tilde{\omega}=0.6$. The inset displays the comparison of exact solution with the numerical simulations. As shown in the inset, our analytical results are in good agreement with the numerical simulations.

(iii) When all higher-order effects in Eq. (7) are considered, i.e., $s_1 \neq 0$, $p_3 \neq 0$, $s_1 \neq 0$, $k_3 \neq 0$, the gray solitary wave possesses the following parameters:

\begin{equation}
\Omega = -\frac{k_3p_3}{12s_2}, \quad (12b)
\end{equation}

\begin{equation}
\chi = \frac{\eta\lambda(k_2 + 23\lambda^2s_2)}{\rho} + \frac{3\eta\lambda(s_1^2 - 4p_3s_2)}{\rho p_5} + \frac{k_3p_3^2}{48s_2^5} - \frac{k_3p_3s_1}{4s_2^3} + \frac{k_3p_3 - 9k_3s_1^2}{12s_2^2} + \frac{5k_3^2p_3\eta}{12p_5s_2} - \frac{(3\lambda^2 + 2\rho^2)k_3p_5 + 6k_3s_1}{12s_2}, \quad (12c)
\end{equation}

\begin{equation}
\eta^2 = -\frac{\rho^2s_1^2}{2k_3}, \quad (11a)
\end{equation}
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FIG. 3. (Color online) The influences of the higher-order effects (a) \( k_3 \) and (b) \( s_1 \) on \( \chi, k, \eta, \) and \( \Omega \) of the gray solitary wave (11) for \( \omega_m/\omega_p=0.9, \, \tilde{\omega}=0.6; \) (c) the distributions of the gray solitary wave (11) for \( \omega_m/\omega_p=0.9 \) when \( \tilde{\omega}=0.65, \) corresponding to \( k_3=-1.0125, \) \( p_3=1.6727 \times 10^{-10}, \) \( k_3=0.8358, \) and \( s_1=-0.2625 \times 10^{-10}; \) and \( \tilde{\omega}=0.6, \) corresponding to \( k_3=-1.3073, \) \( p_3=1.5806 \times 10^{-10}, \) \( k_3=0.9996, \) and \( s_1=-0.3197 \times 10^{-10}. \) (d) The evolution of the gray solitary wave (11) for \( \omega_m/\omega_p=0.9, \, \tilde{\omega}=0.6. \) The inset shows the comparison of the analytical solution (11) (circle) with numerical simulation (dotted line) and initial pulse (solid line). Here, \( \lambda=10^5; \, \rho=10^5, \) and soliton intensities are normalized as in Fig. 2.

\[
k = -\frac{k_3 p_3^3}{1728 s_2^3} + \frac{k_3 p_3^2 s_1}{96 s_2^2} - \frac{k_3 p_3 (k_2 p_3 + 18 s_1^2)}{288 s_2^2} - \frac{\eta k_3 p_3^2}{48 s_2^2} + \frac{\lambda k_3^2 p_3 s_1 \eta}{4 p_3 s_2} + k_3 \left( (2 \lambda^2 + 3 \rho^2) k_3 s_2^3 + 3 k_2 p_3 s_1 + 9 s_1^3 \right) 72 s_2^3 - \frac{\eta k_3 (k_2 p_3 + 9 s_1^2)}{12 p_3 s_2} - \frac{s_1 (k_2 p_3^2 + 2 k_3 p_3^2 + k_3 s_1)}{8 s_2^2} + 12 \lambda k_3 (k_2 p_3^2 + k_3 s_1) 24 s_2 + \frac{\lambda \eta k_3 (k_3 p_3 + 2 k_3 s_1)}{4 p_3 s_2} + \frac{12 \lambda^2 p_3 - (13 \lambda^2 + 2 \lambda^2 \rho^2 + 6 \rho^4) p_3}{12} 3 \lambda^2 + 2 \rho^2 = -\frac{5 k_3^2 p_3^2 + 36 k_3 p_3 s_1 s_2 - 12 k_3 p_3^2 - 36 s_1^2 s_2^2 - 144 p_3 s_2^3}{108 p_3 s_2^3}.
\]

Obviously, \( \eta \) and \( \rho \) are related by Eq. (12a) and \( \lambda \) and \( \rho \) are related by Eq. (12e), so there is one free parameter among them. Meanwhile, Eq. (12a) requires pseudo-\( \chi^{(5)} \) nonlinearity and second-order nonlinear dispersion must satisfy \( s_2 \rho_3 < 0, \) and Eq. (12e) requires \( -5 k_3^2 p_3^2 + 36 k_3 p_3 s_1 s_2 - 12 k_3 p_3^2 - 36 s_1^2 s_2^2 - 144 p_3 s_2^3 < 0. \) These conditions mean that the formation of this gray solitary wave (12) strongly depends on pseudo-\( \chi^{(5)} \) nonlinearity \( p_3 \) and second-order nonlinear dispersion \( s_2. \) It is noted that the formation conditions are relatively strict. According to the relation of model parameters shown in Fig. 1, we find that this gray solitary wave (12) may exist in NIMs. For example, for \( \omega_m/\omega_p=0.9 \) and \( \tilde{\omega}=0.42, \) corresponding to \( k_3=-3.8616, \) \( k_3=2.9176, \) \( p_3=1.1573 \times 10^{-10}, \) \( p_3=-0.061 \) \( 9.8 \times 10^{-10}, \) \( s_1=-0.410 \) \( 38 \times 10^{-10}, \) and \( s_2=0.0571 \) \( 18 \times 10^{-10}, \) the evolution of this gray solitary wave (12) in NIMs is shown in Fig. 4(a) and the corresponding numerical verification is shown in the inset. It is clear that the analytical solution well coincides in the numerical simulation. Furthermore, we investigate the influence of the higher-order effects such as the TOD \( k_3, \) self-steepening \( s_1, \) pseudo-\( \chi^{(5)} \) \( p_3, \) and second-order nonlinear dispersion \( s_2 \) on the width \( \eta, \) dip depth \( \rho, \) velocity \( \chi, \) frequency shift \( \Omega, \) and wave number \( k \) of the solitary wave (12), as shown in Figs. 4(b)–4(e). It should be pointed out that in order to ensure the model parameters located in the negative-index region, we take the values of these higher-order terms according to Fig. 1. Similar to case (ii), these higher-order effects influence much more the velocity \( \chi \) and wave number \( k \) than the width \( \eta, \) dip depth \( \rho, \) and frequency shift \( \Omega. \) Figure 4(f) presents the curves of the output pulses at \( \zeta=0.2 \) at \( \tilde{\omega}=0.5 \) and 0.45 for \( \omega_m/\omega_p=0.9. \) It can be seen from Fig. 4(f) that under the different balances of param-
eters, the gray solitary wave (12) possesses slightly different dip depths, obviously different widths, and remarkably different velocities, which are in agreement with the curves of Figs. 4(b)–4(e). These results show that $\chi^{(3)}$ nonlinearity, pseudo-$\chi^{(5)}$ nonlinearity, self-steepening, and second-order nonlinear dispersion mutually interact and reach a balance to ensure the formation of gray solitary waves; in particular, pseudo-$\chi^{(5)}$ nonlinearity and second-order nonlinear disper-

FIG. 4. (Color online) (a) The evolution of the gray solitary wave (12) for $\omega_m/\omega_p=0.9$, $\bar{\omega}=0.42$; corresponding parameters are specified in the text. Numerical verification is shown in the inset. The influences of the higher-order effects (b) $k_3$, (c) $p_s$, (d) $s_1$, and (e) $s_2$ on $\chi$, $k$, $\eta$, and $\Omega$ of the gray solitary wave (12). (f) The distributions of the gray solitary wave (12) for $\omega_m/\omega_p=0.9$ when $\bar{\omega}=0.45$, $\bar{\omega}=0.5$. Here, $\lambda=10^5$ and soliton intensities are normalized as in Fig. 2.
sion play a crucial role for the existence and properties of this case of gray solitary wave.

**IV. CONCLUSION**

We have derived a higher-order NLS equation governing the propagation of few-cycle electromagnetic pulse in nonlinear NIMs under reasonable approximations. Using analytical method, we have presented three cases of exact gray solitary-wave solutions and the corresponding formation method, we have investigated the properties of these gray solitary waves under the influence of higher-order linear and nonlinear effects. Analytical and numerical results indicated that these exact gray solitary waves can exist in NIMs, and that the higher-order effects not only play an important role for the formation of gray solitary waves but also dominate their propagations in NIMs. The obtained results will be significant for the future studies on ultrashort solitons in the nonlinear NIMs.

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