Combined frequency conversion and pulse compression in nonlinear tapered waveguides

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We suggest an application of pump-degenerate four-wave mixing process in tapered waveguides for generation of ultrashort pulses with central frequency tunable over the material transparency range. Our method can produce strongly compressed frequency-converted pulses in presence of group-velocity mismatch and group-velocity dispersion. Additionally, the proposed technique does not require pulse phase synchronization and effectively operates for strongly chirped pump pulses, thus enabling the use of longer nonlinear media for high conversion efficiency. © 2012 Optical Society of America

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Frequency conversion of ultrashort pulses is an important task in nonlinear optics. Recently developed silicon-on-insulator (SOI) subwavelength waveguides combine enhanced cubic nonlinearity and possibility of on-chip integration, allowing for effective and relatively broadband frequency conversion via four-wave mixing (FWM) [1,2]. Whereas such waveguides could facilitate frequency conversion across all Si material transparency range including midinfrared wavelengths, one of the main limiting factors in case of significantly different pump, signal, and idler frequencies is the group-velocity mismatch (GVM). In combination with group-velocity dispersion (GVD), it may lead to temporal broadening of frequency-converted pulses and reduced conversion efficiency [3]. One common approach to mitigate this is by using short nonlinear samples, however, then for good conversion efficiency high peak power pump is required, which can lead to undesirable nonlinear pulse distortion.

Interestingly, FWM dynamics in media with cubic nonlinearity can have certain similarities to second-harmonic generation (SHG) in quadratic nonlinear media [4]. The pulse compression in the process of SHG can be realized in bulk [5–7] and aperiodically poled nonlinear crystals [8,9]. The latter technique was also applied to optical parametric amplification (OPA) with continuous-wave (CW) seeding, but only in degenerate regime [10]. The main remaining limitation for these methods is that the generation of compressed pulses through SHG at arbitrary reconfigurable wavelengths is not possible. Whereas GVM-induced pulse broadening can also be alleviated for sum-frequency [11,12] and difference-frequency generation [13], which may allow more tunability over the converted pulse wavelength, the required precise synchronization of two pump pulses may be complicated to implement.

In this Letter, we propose a method for generating ultrashort pulses using the process of pump-degenerate FWM in subwavelength tapered waveguides with enhanced cubic nonlinearity. The idler wavelength can be tuned arbitrarily within very broad range by adjusting the frequency detuning of monochromatic signal wave with respect to the pump and choosing the appropriate waveguide tapering. We demonstrate that highly efficient idler-pulse compression can be achieved in subwavelength tapered waveguides using chirped pump pulses, despite large GVM. We show that the process of idler-pulse generation is mathematically analogous to SHG for weak pump and that the pulse compression is increased even further in the regime of high conversion efficiency. Another advantage of the proposed scheme is that it does not require either pumping with synchronized pulses or the phase synchronization between input pump and signal waves.

We study the dynamics of collinear lossless FWM process with degenerate pump [see phase-matching diagram in Fig. 1] and use coupled-wave equations in low pump depletion approximation [2]:

\[
\frac{\partial A_s}{\partial z} + \frac{\delta_{sp}(z)}{\tau} \frac{\partial A_s}{\partial \tau} + \frac{i D_s(z)}{2} \frac{\partial^2 A_s}{\partial \tau^2} = -i \gamma_s(z) A_p^2 A_i^* \exp[iC(z)] - 2i \gamma_s(z) |A_p|^2 A_s, \tag{1}
\]

\[
\frac{\partial A_i}{\partial z} + \frac{\delta_{ip}(z)}{\tau} \frac{\partial A_i}{\partial \tau} + \frac{i D_i(z)}{2} \frac{\partial^2 A_i}{\partial \tau^2} = -i \gamma_i(z) A_p^2 A_s^* \exp[iC(z)] - 2i \gamma_i(z) |A_p|^2 A_i, \tag{2}
\]

\[
\frac{\partial A_p}{\partial z} + \frac{i D_p(z)}{2} \frac{\partial^2 A_p}{\partial \tau^2} = -i \gamma_p(z) |A_p|^2 A_p. \tag{3}
\]

Here \( z \) is the propagation distance, \( \tau \) is the time relative to the pump pulse center. The subscripts \( s, i, \) and \( p \) denote the variables corresponding to the signal, idler, and pump waves, respectively, \( A_{s,i,p} \) are the complex slowly varying field envelopes, normalized such that \( P_{s,i,p} = |A_{s,i,p}|^2 \) represent the optical powers. The coefficients \( \delta \) define the
GVMs between the signal (\(\delta_{s,p}\)) or idler (\(\delta_{i,p}\)) waves and the pump, while coefficients \(D\) define the GVD for the signal (\(D_s\)), idler (\(D_i\)), and pump (\(D_p\)) waves. The \(\gamma_{s,i,p}\) are effective nonlinear coefficients where subscripts \(s\) and \(i\) correspond to FWM for signal and idler and \(p\) to pump self-phase-modulation processes. The function \(C(z) = 2\pi z[2n_p/\lambda_p - n_s/\lambda_s - n_i/\lambda_i]\) defines the phase velocity mismatch between the interacting waves, where \(\lambda_{s,i,p}\) are the central wavelengths.

We now formulate the optimal conditions for the generation of ultrashort idler pulses. In order to reduce the pump self-action and nonlinear absorption, it is beneficial to lower the peak pump power while preserving the high total pulse energy. This requirement is satisfied for chirped pulses. Specifically, we consider a chirped gaussian pump pulse with the input profile: \(A_p(0,\tau) = P_p^{1/2}\exp[-\tau^2/(2\tau_0^2 + 2iC_p^2)]\), where \(P_p\) is the peak input power, \(C_p\) is a pulse chirp coefficient, and \(\tau_0\) corresponds to full width half maximum of nonchirped pump pulse. As the signal input, we take small-amplitude CW field (or a long narrow-band pulse) and consider zero idler amplitude at the input, see Fig. 1. In this approach, the idler-pulse wavelength can be tuned by choosing the pump and signal wavelengths, as \(\lambda_i^* = 2\lambda_s^* - \lambda_p^*\). Whereas a similar conversion scheme was previously developed for optical gating in homogeneous waveguides [4], we demonstrate below that idler-pulse can be compressed in a waveguide with specially designed tapering such that the phase-matching condition varies along the waveguide as follows:

\[
C(z) = C_w[z - L/2]^2. \tag{4}
\]

Here \(L\) is the total waveguide length and \(C_w\) is the effective tapering coefficient, which value should be optimized to achieve maximum pulse compression.

We first derive an analytical expression for the optimal value \(C_w\) under certain simplifications. We assume GVM and nonlinearity to be practically constant along the waveguide, \(\delta_{s,i,p}(z) = \delta_{s,i,p}\) and \(\gamma_{s,i,p}(z) = \gamma_{s,i,p}\), and neglect GVD, \(D_s,i,p = 0\). We also consider the regime of low conversion, when the pump power is rather small such that \(A_p \gg A_s \gg A_i\) along the waveguide. Then, the nonlinear terms in Eq. (1) and Eq. (3) can be neglected, and there is almost no temporal reshaping of input pump pulse and CW signal. Finally, Eq. (2) can be reduced to \(\partial A_i/\partial z + \delta_{p} A_s A_i/\partial \tau = -i\gamma A_s^2 A_i^* \exp[C(z)]\). Since under our assumptions \(A_s\) remains a CW signal, this equation has the same form as for the second-harmonic wave in the SHG process in the nondepleted pump approximation where \(C(z)\) can be engineered through crystal poling [9]. We then adopt the results for pulse compression with SHG [9] and determine the optimal pump chirp as

\[
C_p = \delta_{i,p} C_w^{1/2}. \tag{5}
\]

We have checked by direct numerical modeling of Eqs. (1–3) that our approach can be applied to a range of different platforms and wavelengths. As an example, we demonstrate below the plausibility of experimental implementation with SOI platform for \(\lambda_s = 3.6\ \mu m, \lambda_i = 2.16\ \mu m, \lambda_p = 2.7\ \mu m\), noting that in mid-IR wavelength range the model assumptions of no two-photon absorption and no free-carrier generation are valid for both subnanosecond and picosecond pulses [14]. We consider a rectangular cross-section Si waveguide on a SiO2 substrate with length \(L = 3\ cm\), height \(h = 1.2\ \mu m\), and width linearly tapered from \(w = 740\ \mu m\) at the input facet to \(w = 760\ \mu m\) at the output facet. We study the waveguiding dispersion properties using COMSOL RF module and find that the phase mismatch for the pump-degenerate FWM \(C(z)\) incorporating TM signal and pump modes and TE idler mode is equal to zero in the center of the waveguide, i.e. \(C(L/2) = 0\). We furthermore observe that the dependence of the phase mismatch \(C(z)\) matches very closely the optimal dependence in Eq. (4) with tapering coefficient \(C_w = -0.26\ mm^{-2}\). The other dispersion parameters exhibit small linear changes with \(z\): \(\delta_{p}(z) = 2.48 - 0.16z/L\ [ns/m], \delta_{p}(z) = -1.39 + 0.14z/L[ns/m], D_s(z) = -24.5 + 1.12/L[ps^2/m], D_i(z) = -0.95 + 0.02z/L[ps^2/m], D_p(z) = -7.45 + 0.5z/L[ps^2/m]\). Since cubic nonlinearity for wave mixing involving different frequencies is not yet characterized for Si in mid-IR region, we make an order-of-magnitude estimate for the nonlinear coefficients as \(\gamma_{s,i,p} = 2\gamma_{s,i,p}(C_w^{1/2})\), with material nonlinearity \(n_2 = 6 \cdot 10^{-5}\ cm^2/GW\ [2]\) and the effective area \(a_{eff} = 0.75\ \mu m^2\). We check that the changes of \(\gamma_{s,i,p}\) within one order of magnitude do not qualitatively affect the results. The effective area for all modes changes with \(z\) by less than 2% and therefore we neglect this change. We consider an input chirped gaussian pump pulse with \(P_p = 100\ mW, \tau_0 = 1\ ps\), and the chirp according to Eq. (5) as \(C_p = \delta_{i,p}(L/2)/C_w^{1/2} = 2.58\ ps\). The overall length of the pump pulse is then \((2^{1/2}\tau_0^2 + (2^{1/2}C_p^{1/2}/\tau_0)^{21/2}) = 9.44\ ps\). The initial power of CW signal is \(P_s = 1\ mW\) and there is no idler input. We note here, that unlike degenerate OPA [10], the phase synchronization between pump and signal waves is not required for pump-degenerate FWM, as it is automatically compensated by the idler phase.

Now we demonstrate the results of numerical simulation with Eqs. (1–3) and parameters for SOI waveguide provided above. The dynamics of normalized idler-pulse, spectrum, and peak power along the propagation are shown in Figs. 2(a–c). We see that at the initial propagation stage up to \(z = 1\ cm\) [Fig. 2(a)], the idler-pulse goes through unstable transition where the effects of non-phase-matched generation from the center of the pump pulse and phase-matched generation from the low-intensity pump pulse lobe compete. It happens because in chirped pump pulse the wavelengths are distributed along its duration and only those at the beginning of the pulse are phase-matched in the initial section of the tapered waveguide. At \(z = 1\ cm\) the idler wave is now generated effectively at phase-matched wavelength and thus idler-pulse compression begins and continues throughout the taper. During this process different idler frequency components are generated along the waveguide [Fig. 2(b)] and are effectively assembled together by GVM in temporal domain [Fig. 2(a)]. The idler peak power grows [Fig. 2(c)] until the idler spectrum width becomes similar to that of the pump and idler-pulse is compressed to \(\tau_0 = 1\ ps\). At that point the idler power
reaches 0.4 µW, the signal power is increased to 4.9 mW, and the pump power stays unchanged, which means that the relation $A_p \gg A_s \gg A_i$, which was used to derive Eq. (5), remains satisfied.

We now compare these results with nontapered waveguide of constant width $w = 750$ nm corresponding to the exact phase-matching of the central wavelengths, see Figs. 2(d–f). We see in Fig. 2(d) that initially there is some degree of pulse compression in analogy with pulse conversion in quadratic media [6,11,12]. However at $z = 5$ mm the pulse begins to spread, while its spectrum is narrowed [Fig. 2(e)] and its peak power no longer grows [Fig. 2(f)]. Even if we consider the length $L$ equal to the optimal value $z = 5$ mm, the nontapered waveguide still provides four times less pulse compression and four times less conversion efficiency compared to the tapered structure.

Finally we demonstrate generation of compressed idler-pulse in the tapered waveguide with higher conversion efficiency. As we increase the pump peak power from $P_p = 100$ mW to $P_p = 5$ W, the nonlinear interaction starts to change both the signal and idler profiles and the condition $A_s \gg A_i$ no longer applies. Simulations show that in this regime, the idler peak power grows much faster than the signal power and reaches 15 mW, while the signal power is increased only to 7 mW. As a result the idler-pulse compression occurs faster, spectrum becomes broader, and the compression ratio is enhanced by 10% in comparison to the low pump power case.

We note that in the provided examples the GVD played a relatively minor role. However both the linear tapering and GVD lead to linear chirp (or compensation of linear chirp) in frequency-converted pulse. Therefore they can compensate each other even in the case of very large GVD. Mitigation of higher orders of dispersion, as well as studying highly tapered waveguides with stronger dependence of GVM and GVD on propagation distance, may be of interest for the future research.

In conclusion, we have suggested a method for efficient frequency conversion combined with pulse compression based on nonlinear FWM process with pre-chirped pump pulses. This approach can be realized in tapered subwavelength waveguides. We anticipate that the proposed method will find applications for efficient ultrashort pulse frequency conversion.

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References