Two-color surface lattice solitons

Zhiyong Xu* and Yuri S. Kivshar

Nonlinear Physics Center and Center for Ultra-high Bandwidth Devices for Optical Systems (CUDOS), Research School of Physical Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia
**Corresponding author: xzy124@rsphysse.anu.edu.au*

Received July 18, 2008; revised September 17, 2008; accepted September 18, 2008;
posted October 7, 2008 (Doc. ID 99072); published October 30, 2008

We study the properties of surface solitons generated at the edge of a semi-infinite photonic lattice in nonlinear quadratic media, namely two-color surface lattice solitons. We analyze the impact of phase mismatch on the existence and stability of nonlinear surface modes and find novel classes of two-color twisted surface solitons, which are stable in a large domain of their existence. © 2008 Optical Society of America

OCIS codes: 190.0190, 190.6135.

Surface modes appear as a special type of waves localized near an interface separating two different media. In optics, linear electromagnetic surface waves are known to exist at an interface separating homogeneous and periodic dielectric media [1]. Recently, the interest in the study of electromagnetic surface waves has been renewed, and it was shown theoretically [2–4] and experimentally [5–8] that nonlinearity-induced self-trapping of light may become possible near the edge of a one-dimensional waveguide array leading to the formation of discrete surface solitons (see also the review papers [9,10]). In particular, it was found that the self-trapped surface modes acquire some novel properties different from those of the discrete solitons in infinite lattices: discrete surface states can exist only above a certain threshold power and, for the same value of the power, up to two different surface modes can exist simultaneously. This can be understood as discrete optical solitons [11] localized near the surface and the action of a repulsive force from the surface [3].

Surface solitons are usually studied for cubic or saturable nonlinear media. However, Siviloglou *et al.* [6] reported on the first observation of discrete quadratic surface solitons in periodically poled lithium niobate waveguide arrays. By operating on either side of the phase-matching condition and using the cascading nonlinearity, they observed both in-phase and staggered discrete surface solitons. They also employed a discrete model with decoupled waveguides at the second harmonics to model some of the effects observed experimentally.

The purpose of this Letter is twofold. First, we extend substantially the earlier theoretical analysis performed by Siviloglou *et al.* [6] and study, for the first time to our knowledge, two-color quadratic surface solitons in a continuum model with a truncated periodic potential. We analyze the effect of the mismatch on the existence, stability, and generation of surface solitons located at the edge of a semi-infinite waveguide array in nonlinear quadratic media. Second, we reveal the existence of novel classes of two-color surface solitons that are stable in a large domain of their existence.

We consider propagation of light at the interface of a semi-infinite lattice imprinted in quadratic nonlinear media, which involves the interaction between fundamental frequency (FF) and second-harmonic (SH) waves. Light propagation is described by the following coupled nonlinear equations [12,13]:

$$i\frac{\partial q_1}{\partial z}=\frac{d_1}{2}\frac{\partial^2 q_1}{\partial x^2}-q_1^* q_2\exp(-i\beta z)-pR(x)q_1,$$

$$i\frac{\partial q_2}{\partial z}=\frac{d_2}{2}\frac{\partial^2 q_2}{\partial x^2}-q_1^2\exp(i\beta z)-2pR(x)q_2, \qquad (1)$$

where q_1 and q_2 represent the normalized complex amplitudes of the FF and SH fields; x and z stand for the normalized transverse and longitudinal coordinates, respectively; β is the phase mismatch; $d_1=-1$, $d_2=-0.5$; p is the lattice depth; and the function $R(x)=0$ at $x<0$ and $R(x)=1-\cos(Kx)$ at $x\geqslant 0$ describes the profile of a truncated periodic lattice with modulation K. In typical quadratic nonlinear crystals, for a beam width of $\sim 15\ \mu$m, the distance z in the range 0–30 corresponds to a few centimeters, and the peak intensities will be in the range of $0.1-10\ \mathrm{GW/cm^2}$ for the formation of lattice solitons at wavelengths $\lambda=1\ \mu$m; a refractive index modulation depth of the order of 10^{-4} corresponds to the lattice depth $p\sim 1$ [12]. The system of Eq. (1) admits several conserved quantities including the power $P=\int_{-\infty}^{\infty}(|q_1|^2+|q_2|^2)\mathrm{d}x$.

The stationary solutions for the lattice-supported surface solitons can be found in the form $q_{1,2}(x,z)=u_{1,2}(x)\exp(ib_{1,2}z)$, where $u_{1,2}(x)$ are real functions and $b_{1,2}$ are real propagation constants satisfying $b_2=\beta+2b_1$. Families of surface solitons are determined by the propagation constant b_1, the lattice depth p, and the phase mismatch β. For simplicity, we set the modulation parameter $K=4$. To analyze stability we examine perturbed solutions $q_{1,2}(x,z)=[u_{1,2}(x)+U_{1,2}(x,z)+iV_{1,2}(x,z)]\exp(ib_1z)$, where real parts $U_{1,2}$ and imaginary parts $V_{1,2}$ of perturbation can grow with the complex rate δ. Linearization of Eq. (1) around $u_{1,2}$ yields the eigenvalue problem

$$\delta U_1=\frac{d_1}{2}\frac{\partial^2 V_1}{\partial x^2}-(u_1V_2-u_2V_1)-pRV_1+b_1V_1,$$

$$\delta V_1 = -\frac{d_1}{2}\frac{\partial^2 U_1}{\partial x^2} + (u_1 U_2 + u_2 U_1) + pRU_1 - b_1 U_1,$$

$$\delta U_2 = \frac{d_2}{2}\frac{\partial^2 V_2}{\partial x^2} - 2u_1 V_1 - 2pRV_2 + b_2 V_2,$$

$$\delta V_2 = -\frac{d_2}{2}\frac{\partial^2 U_2}{\partial x^2} + 2u_1 U_1 + 2pRU_2 - b_2 U_2, \qquad (2)$$

which we solve numerically to find the growth rate δ.

Figures 1(a) and 1(b) show the examples of the simplest two-color surface lattice solitons, the so-called odd solitons. Such surface modes reside in the first lattice channel, where both FF and SH fields reach their maxima. Owing to the lattice truncation, the solitons have asymmetric profiles at lower power [Fig. 1(a)]. There exists a lower cutoff (b_{co}) of the propagation constant for the existence of odd solitons. The power of odd solitons is a nonmonotonic function of the propagation constant and there is a narrow region close to the cutoff where the power dependence changes its slope, $\mathrm{d}P/\mathrm{d}b_1 < 0$ [Fig. 2(a)]. We find that the cutoff b_{co} is a monotonically increasing function of lattice depth p, and it is a decreasing function of the phase mismatch [Fig. 2(b)]. It should be noted that the critical power for the existence of two-color surface lattice solitons depends on the phase mismatch; it reaches the minimum value for the exact phase matching ($\beta=0$). Linear stability analysis reveals that these odd surface solitons are stable almost in the whole domain of their existence except a very narrow region near the cutoff b_{co} where the exponential instability develops. Direct numerical simulations of the model [Eq. (1)] confirm the results of the linear stability analysis.

In addition to the simplest solitons described above, we find various families of higher-order surface lattice solitons, which can be viewed as the combination of several in-phase and out-of-phase odd solitons. Here we focus only on the case where the FF field features the out-of phase combinations because the modes with in-phase combinations are unstable. Being reminiscent to its discrete counterparts such as twisted localized modes, here we term the mode with the out-of phase combination as the *twisted surface lattice soliton*. Typical profiles of twisted modes residing at the edge of a semi-infinite lattice are shown in Figs. 1(c) and 1(d). Similar to the properties of odd solitons, the power of twisted surface solitons is a nonmonotonic function of the propagation constant and there exists a narrow region close to the cutoff where $\mathrm{d}P/\mathrm{d}b_1 < 0$ [Fig. 2(c)]. However, one should note that the cutoff b_{co} is a nonmonotonic function of the lattice depth p, as shown in Fig. 2(d). Likewise, the critical power for the existence of twisted solitons reaches the minimum value at $\beta=0$.

Importantly, we find that the twisted surface lattice solitons become completely stable when the power exceeds a certain threshold value [Fig. 3(a)], while the instability domain expands with the growth of the absolute value of phase mismatch β.

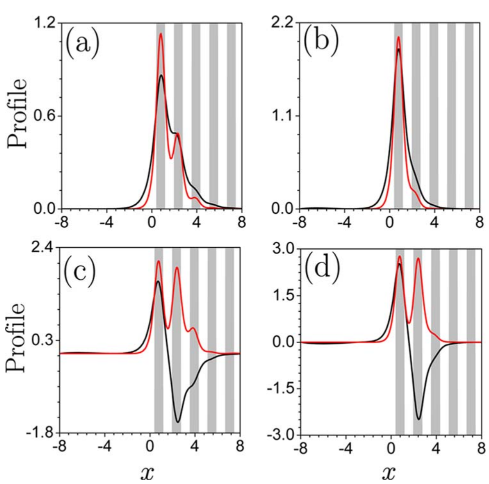

Fig. 1. (Color online) (a),(b) Profiles of two-color odd surface solitons with b_1= (a) 1.5 and (b) 2. (c),(d) Profiles of two-color twisted surface solitons with b_1= (c) 1.75 and (d) 2.2. Black and gray curves show the profiles of FF and SH fields, respectively. Lattice depth $p=1$, and phase mismatch $\beta=0$. In the white region $R(x)<1$, while in the gray region $R(x)\geqslant 1$.

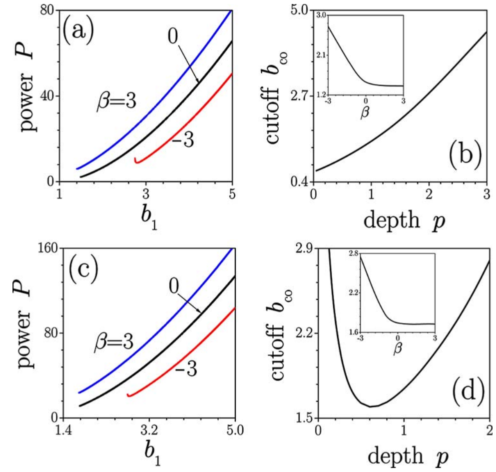

Fig. 2. (Color online) (a) Power of surface solitons versus propagation constant for different phase mismatches. (b) Cutoff of the propagation constant b_1 for surface solitons as a function of depth at $\beta=0$. Inset shows the cutoff versus phase mismatch at $p=1$. (c) Power of twisted surface solitons versus b_1 for different phase mismatches. (d) Cutoff of the propagation constant for twisted surface solitons as a function of lattice depth at $\beta=0$. Inset shows the cutoff versus the phase mismatch at $p=1$.

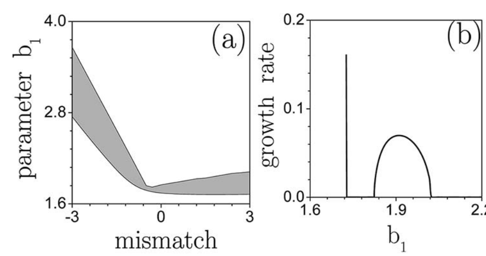

Fig. 3. (a) Instability domain (shaded) for twisted surface solitons on the (b_1, β) plane at $p=1$. (b) Real part of the instability growth rate for twisted surface solitons versus propagation constant at $\beta=0$ and $p=0.5$.

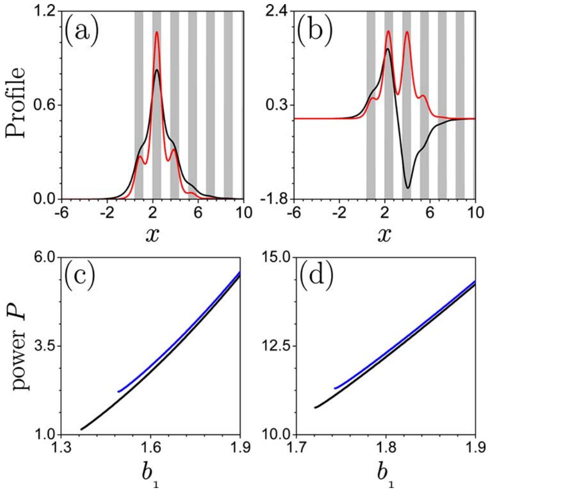

Fig. 4. (Color online) Profiles of two-color surface solitons in the second channel: (a) odd one with $b_1=1.5$ and (b) twisted one with $b_1=1.75$. The black and gray curves show the profiles of FF and SH fields, respectively. Power of (c) odd and (d) twisted surface solitons in the first (upper curve) and second (lower curve) channel of the lattices. Here lattice depth $p=1$ and phase matching $\beta=0$.

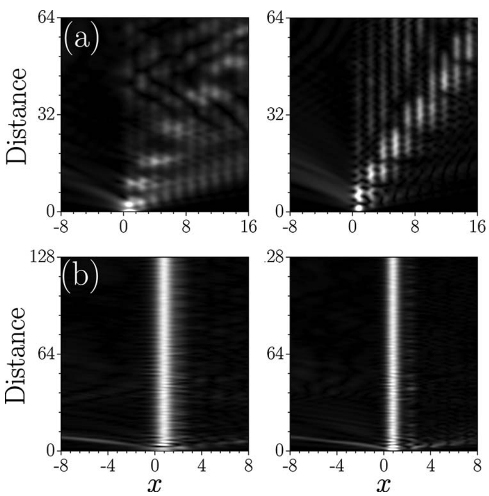

Fig. 5. Excitation of two-color surface solitons from Gaussian input beams: (a) $a_1=1.5$, $a_2=0$; (b) $a_1=2.8$, $a_2=0$. Here lattice depth $p=1$ and phase matching $\beta=0$. Left and right columns show the FF and SH fields, respectively.

Figure 3(b) shows the real part of the perturbation growth rate versus the propagation constant, where the left part corresponds to the exponential instability, while in the right part solitons suffer from oscillatory instabilities. Results from linear stability analysis are confirmed by the direct numerical simulation of the model [Eq. (1)].

We also study the properties of two-color surface modes that are shifted from the lattice edge, similar to the analysis performed earlier for the discrete cubic model [3]. Figures 4(a) and 4(b) show some illustrative examples of odd and twisted states residing in the second channel of the semi-infinite waveguides from the interface. Such modes require much lower critical power compared with surface modes residing at the interface [Figs. 4(c) and 4(d)]. Linear stability analysis reveals that two-color surface solitons in the second channel are stable when their power exceeds a certain threshold.

To address the issue of excitation of two-color surface solitons, we perform a comprehensive study of the soliton generation by two Gaussian beams $q_{1,2}=a_{1,2}\exp(-x^2)$. As shown in the two examples in Figs. 5(a) and 5(b), for lower powers the beams just experience repulsion from the surface and diffraction [Fig. 5(a)]. For high enough powers, surface solitons can be excited either from only the FF field [Fig. 5(b)] or both FF and SH fields (not shown) with a proper phase matching. Two-color surface solitons can be formed for other conditions ($\beta\neq 0$, $a_2\neq 0$) but for higher powers.

In conclusion, we have analyzed the existence, stability, and generation of two-color quadratic surface solitons in a continuum model with a truncated periodic potential and also revealed the existence of novel classes of stable parametrically coupled surface states.

We acknowledge the support of the Australian Research Council and useful discussions with O. Egorov.

References

1. P. Yeh, A. Yariv, and A. Y. Cho, Appl. Phys. Lett. **32**, 104 (1978).
2. K. G. Makris, S. Suntsov, D. N. Christodoulides, G. I. Stegeman, and A. Hache, Opt. Lett. **30**, 2466 (2005).
3. M. Molina, R. Vicencio, and Yu. S. Kivshar, Opt. Lett. **31**, 1693 (2006).
4. Ya. V. Kartashov, V. V. Vysloukh, and L. Torner, Phys. Rev. Lett. **96**, 073901 (2006).
5. S. Suntsov, K. G. Makris, D. N. Christodoulides, G. I. Stegeman, A. Naché, R. Morandotti, H. Yang, G. Salamo, and M. Sorel, Phys. Rev. Lett. **96**, 063901 (2006).
6. G. A. Siviloglou, K. G. Makris, R. Iwanow, R. Schiek, D. N. Christodoulides, G. I. Stegeman, Y. Ming, and W. Sohler, Opt. Express **14**, 5508 (2006).
7. E. Smirnov, M. Stepic, C. E. Ruter, D. Kip, and V. Shandarov, Opt. Lett. **31**, 2338 (2006).
8. C. R. Rosberg, D. N. Neshev, W. Krolikowski, A. Mitchell, R. A. Vicencio, M. I. Molina, and Yu. S. Kivshar, Phys. Rev. Lett. **97**, 083901 (2006).
9. S. Suntsov, K. G. Makris, G. A. Siviloglou, B. Iwanow, R. Schiek, D. N. Christodoulides, G. I. Stegeman, R. Morandotti, H. Yang, G. Salamo, M. Volatier, V. Aimez, R. Ares, M. Sorel, Y. Min, W. Sohler, X. Wang, A. Bezryadina, and Z. Chen, J. Nonlinear Opt. Phys. Mater. **16**, 401 (2007).
10. Yu. S. Kivshar, Laser Phys. Lett. **5**, 703 (2008).
11. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons* (Academic, 2003).
12. Y. V. Kartashov, L. Torner, and V. A. Vysloukh, Opt. Lett. **29**, 1117 (2004).
13. Z. Y. Xu, Y. V. Kartashov, L.-C. Crasovan, D. Mihalache, and L. Torner, Phys. Rev. E **71**, 016616 (2005).