Spiraling multivortex solitons in nonlocal nonlinear media

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Received October 9, 2007; revised December 6, 2007; accepted December 6, 2007; posted December 11, 2007 (Doc. ID 88407); published January 14, 2008

We demonstrate the existence of a broad class of higher-order rotating spatial solitons in nonlocal nonlinear media. We employ the generalized Hermite–Laguerre–Gaussian ansatz for constructing multivortex soliton solutions and study numerically their dynamics and stability. We discuss in detail the tripole soliton carrying two spiraling phase dislocations, or self-trapped optical vortices. © 2008 Optical Society of America

OCIS codes: 190.5940, 190.6135.

Over the past few years nonlocality has proved to have a great impact on both stabilization and propagation dynamics of self-trapped optical beams. Much attention has been given to *nonlocal spatial solitons*, including theoretical studies [1] and experimental demonstrations in nematic liquid crystals [2,3], lead glasses [4,5], and liquids [6]. Recent experimental results [7] for thermal media revealed the existence of multisoliton bound states in the form of dipole, multipole, and necklace beams. In addition, the existence of more complex structures with nonzero angular momenta such as vortex solitons [8–10] and rotating azimuthons [11–13] suggested that nonlocality can support higher-order spiraling solitons.

Recently, we have introduced the so-called Laguerre-nonlocal (LN_{nm}) and Hermite-nonlocal (HN_{nm}) spatial solitons [14], resembling soliton "necklaces" and "matrices." Our theoretical approach is based on an analogy with linear Laguerre–Gaussian (LG) and Hermite–Gaussian (HG) optical modes. In nonlocal media, however, such beams exhibit nontrivial transformations between the structures of different symmetries [14–16]. Similar transformations in thermal media have been reported for multipole vector solitons [17], which suggests that such phenomena are generic and independent of the model. Moreover, during the symmetry transformations we could clearly distinguish intermediate states resembling the generalized Hermite–Laguerre–Gaussian (HLG) modes [18]. This observation indicates that the higher-order self-trapped states may represent structurally stable localized solutions that belong to a general class of solitons with more complex structure and phase.

In this Letter we study spatial nonlocal solitons that realize, in a simple way, a continuous transformation between the HG and LG self-trapped modes through a single parameter α. More specifically, the two symmetries are recovered for the limiting values of this parameter ($0 \leqslant \alpha \leqslant \pi/4$) [18] with all intermediate states remaining structurally stable. First, we construct higher-order solitons that extend the HLG linear modes to the case of nonlinear media with the Gaussian nonlocal response. Second, as a specific example, we consider a tripole-mode soliton with nonzero angular momentum and study its propagation dynamics. We observe stable spatial rotation (spiraling) of the tripole soliton carrying two optical vortices of the same charge. Our theory allows one to estimate the rotation velocity, i.e., the pitch of the helix formed by the braided vortex lines, and to reveal the underlying physical mechanism responsible for this effect.

In nonlocal nonlinear media the propagation of a paraxial optical beam with scalar field envelope E is governed by the nonlinear Schrödinger equation [19]

$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + E\int R(|\mathbf{r}-\mathbf{r}'|)|E(\mathbf{r}')|^2 \mathrm{d}\mathbf{r}' = 0, \tag{1}$$

where for the response function we take $R(t)=\exp(-t^2/\sigma^2)/\pi\sigma^2$, with σ being the degree of nonlocality, and the transverse coordinates are measured in units of σ (see, e.g., [14] for details).

We look for stationary solutions parameterized with propagation constant k, $E=U\exp(ikz)$, where $U(x,y;\alpha)$ is described by the HLG_{nm} modes [18]:

$$U_{nm}(x,y;\alpha) = A\exp(-x^2/2a_x^2 - y^2/2a_y^2) \times \sum_{j=0}^{n+m} C_{nm}^{j}(\alpha) H_{n+m-j}(x/b_x) H_j(y/b_y). \tag{2}$$

Here A is the soliton amplitude, H_j denote Hermite polynomials, and the coefficients $C_{nm}^{j}(\alpha)$ are expressed in terms of the Jacobi polynomials $P_j^{u,v}$ as follows: $C_{nm}^{j}(\alpha)=i^j \cos^{n-j}\alpha \sin^{m-j}\alpha P_k^{(n-k,m-j)}(-\cos 2\alpha)$.

To derive approximate stationary solutions we employ a standard variational approach with the ansatz given by Eq. (2). For this work, we concentrate on a particular solution of this kind, the tripole soliton [7] with an additional simplification of $a_{x,y} \equiv a$ and $b_{x,y} \equiv b$. As a result, the tripole soliton, i.e., a structure consisting of three in-line peaks separated by the zeros of intensity [see Fig. 1(a)], corresponds to the

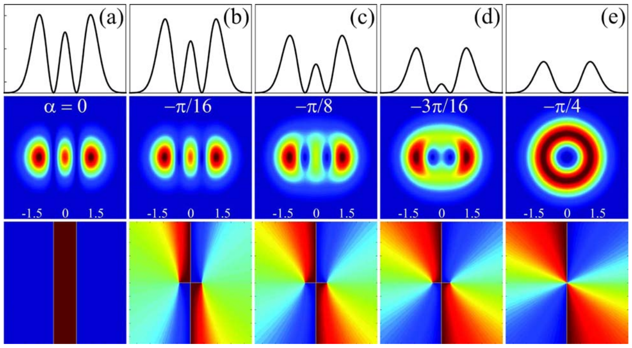

Fig. 1. (Color online) Family of the tripole-like solitons for power $P=200$ versus α. The cross sections of the beam profiles are shown in the top row. The colors for intensity (middle row) are mapped from zero (blue online) to maximal value (red online), while phase (bottom row) is scaled from $-\pi$ to π.

mode $U_{20}(x,y;\alpha=0)$ and, according to Eq. (2), it can be deformed continuously to a double-charge vortex beam:

$$U_{20}(x,y;\alpha) = A\exp(-(x^2+y^2)/2a^2) \times [4(x\cos\alpha + iy\sin\alpha)^2/b^2 - \cos 2\alpha]. \quad (3)$$

A complex conjugation or, alternatively, reflection $\alpha \to -\alpha$, produces a vortex of the opposite charge.

Varying α is equivalent to the astigmatic transformation from the HN_{20} to LN_{20} mode [18]. We obtain numerically the variational parameters for different values of α and k and construct the soliton families as shown in the example of Fig. 1. For $\alpha=0$ the solution has a relatively simple phase structure of the HG_{20} mode. More general tripole-like structures appear at the intermediate values of α. In this case Figs. 1(b)–1(d) clearly show two separate vortices with the same topological charge embedded into the beam. At $|\alpha|=\pi/4$ these vortices merge to form a double-charge vortex ring LN_{20}.

As the next step, we employ variational solutions as an input for direct simulations of Eq. (1) using the fast-Fourier-transform split-step beam propagation algorithm. We perform extensive numerical simulations to study the dynamics and stability properties of the tripole beams and observe spiraling of tripole beams with nonzero angular momentum ($\alpha \neq 0$) in nonlocal media. This behavior is in contrast with the dynamics of the corresponding linear modes, which do not rotate [18]. To resolve this controversy, we follow an approach developed by Rozanov [20], which enables finding the rotation velocity Ω of localized beams, provided the solutions of the original propagation equation are known. By using our variational solutions, Ω can be represented as a sum of the linear (Ω_{ln}) and nonlinear (Ω_{nln}) parts,

$$\Omega = \Omega_{\mathrm{ln}} + A^2\Omega_{\mathrm{nln}}, \quad (4)$$

where $\Omega_{\mathrm{ln}} = \gamma \sin 2\alpha(8a^4 - 6a^2b^2 + b^4)$. The expressions for $\Omega_{\mathrm{nln}}(a,b;\alpha)$ and $\gamma(a,b;\alpha)>0$ are too cumbersome to be shown here; note that inverting the sign of α reflects $\Omega \to -\Omega$. Importantly, in the limit $b \to a\sqrt{2}$, i.e., when $U(x,y,\alpha)$ is exactly an eigenmode of the linear equation, Ω_{ln} vanishes, as expected in accordance with [18]. Hence, it is precisely the deformation $b \neq a\sqrt{2}$ that initiates the rotation of the beam even in free propagation. Indeed, such deformed beams can be seen as a superposition of several HLG modes, and the rotation appears as a beating between the modes. However, such beams always rotate in nonlinear media because the nonlinear component Ω_{nln} does not vanish even for the linear modes with $b=a\sqrt{2}$. Note the similar effect of the enhanced rotation of two dark vortices in defocusing media [21] with respect to their spiraling in free propagation [22].

Using Eq. (4) and the variational solutions for A, a, and b, we calculate Ω for different tripole solitons in the parameter domain (k,α); see Fig. 2(a). Surprisingly, for positive angular momentum ($\alpha \geqslant 0$), we obtain negative angular velocity for small k and $\alpha > \pi/5$. However, at such small values of k (and low power $P=\int |E(\mathbf{r})|^2 d\mathbf{r}$ the tripole beams are unstable and, similar to the case of local media, they break up into several fundamental solitons. Since our approximate soliton profiles differ from the exact stationary solutions, the beams oscillate during their propagation, facilitating their break up at low powers. In contrast, for the powers above a certain threshold the tripole beams become stable, surviving strong oscillations over several hundreds of the soliton periods $\sim \pi/k$. In Fig. 2(b) we show the stability domain in the plane (α,P). In contrast with the case of rotating nonlocal dipole [16], where the stability threshold monotonically increases with angular intensity modulation, here the threshold attains its maximum for $\pi/8 < \alpha < 3\pi/8$. In both cases (dipole and tripole), the power threshold attains its minimum for the radially symmetric vortex ring, $\alpha=\pi/4$.

Increasing power in our scaled system is equivalent to the transition to highly nonlocal regime, $\sigma \to \infty$, where the nonlinear term in Eq. (1) can be reduced to an effective harmonic potential:

$$\int R(|\mathbf{r}-\mathbf{r}'|)|E(\mathbf{r}')|^2 \mathrm{d}\mathbf{r}' \to (1 - r^2/\sigma^2)P/\pi\sigma^2. \quad (5)$$

Equation (1) then becomes linear, and it has exact solutions in the form of stationary HLG modes (2) with $b=a\sqrt{2}$ and zero angular velocity, $\Omega=0$. In good agreement with this transition, our full model predicts a very slow decrease of Ω at high powers and $k>100$, not shown in Fig. 2(a), i.e., solitons with larger "mass" P rotate slower.

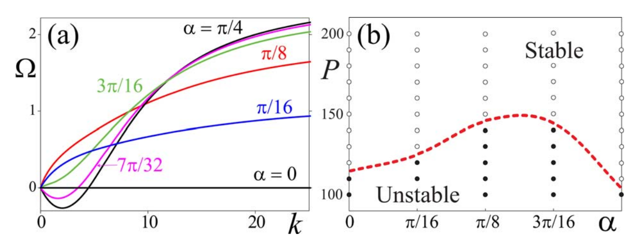

Fig. 2. (Color online) (a) Rotation velocity (4) of the tripole solitons. (b) Stability domain in the parameter plane (α,P).

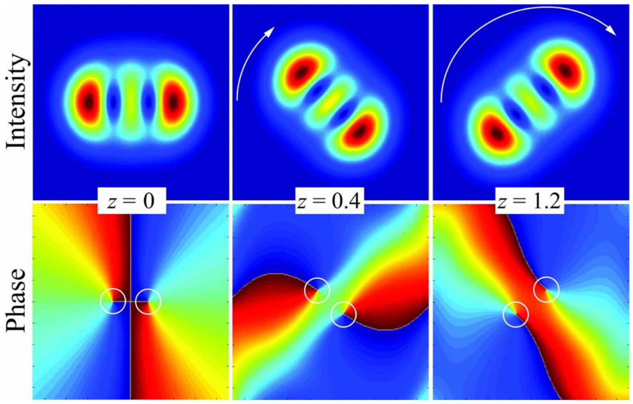

Fig. 3. (Color online) Propagation dynamics of the tripole with $\alpha=-\pi/8$, $k=89.4$, and $P=200$. The corresponding variational solution predicts $\Omega=1.89$, which corresponds to the period of rotation (pitch) $\simeq 3.33$; the numerically obtained values are very close, pitch 3.03 and $\Omega \simeq 2.07$.

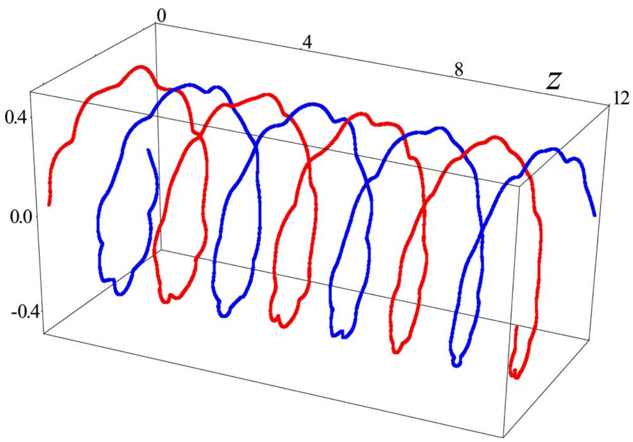

Fig. 4. (Color online) Double helix formed by the braided nodal lines obtained by tracing the spatial locations of the vortex cores during propagation in Fig. 3.

To illustrate stable spiraling of the higher-order nonlocal beams, in Fig. 3 we show the intensity and phase distributions of the tripole soliton with $\alpha=-\pi/8$ and $P=200$. The direction of rotation is indicated by white arrows, while the spatial locations of two vortices embedded in the phase distribution are marked by white circles. Previous theoretical and experimental studies demonstrated unstable spiraling of *bright solitons* [23] and *dark vortices* [21], but, to the best of our knowledge, here we present the first example of stable spiraling of self-trapped vortices in self-focusing nonlocal media. We did not observe any mutual transformations between the different modes [14,16] since such transitions are forbidden due to the conservation of the angular momentum.

Tracing the spatial positions of the vortex cores, in Fig. 4 we reconstruct a double helix formed by the braided nodal lines. An analogous effect in free propagation in linear media can be observed by perturbing a specific superposition of nondiffracting Bessel beams with a plane wave [24].

In conclusion, we have studied multivortex beams in nonlocal nonlinear media. As a specific example, we have analyzed in detail a tripole-like soliton carrying a pair of vortices. The general approach developed here can be useful for analyzing other types of singular self-trapped beams, i.e., the beams with nonzero angular momentum carrying optical vortices that may describe knots and links of vortices [25] in nonlinear media.

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