# Fano resonance in quadratic waveguide arrays

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We study resonant light scattering in arrays of channel optical waveguides in which tunable quadratic nonlinearity is introduced as nonlinear defects by periodic poling of single (or several) waveguides in the array. We describe novel features of wave scattering that can be observed in this structure and show that it is a good candidate for observation of Fano resonance in nonlinear optics. © 2005 Optical Society of America

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The study of nonlinear dynamics and spatial solitons in optical systems recently attracted a great deal of attention.[1] In particular, many specific properties of nonlinear lattice systems can be analyzed for arrays of weakly coupled optical waveguides in which both nonlinearity and diffraction may differ dramatically from those in corresponding continuous systems.[2,3]

During the past few years there has been growing interest in the study of nonlinear optics associated with so-called quadratic nonlinearities that may produce effects resembling those known to occur in cubic nonlinear materials. Typical examples are all-optical switching phenomena in interferometric or coupler configurations as well as the formation of spatial and temporal solitons in planar waveguides.[4] Recently it was demonstrated experimentally[5] that arrays of coupled channel waveguides fabricated in a periodically poled lithium niobate slab represent a convenient system with which to verify many theoretical predictions experimentally, including observation of two-frequency discrete solitons mutually locked by quadratic nonlinearity. These experimental observations open many perspectives for employing larger nonlinearities in lattice systems made from quadratic materials.

In this Letter we suggest that arrays of weakly coupled nonlinear quadratic waveguides may be employed for the study of novel effects in resonant light scattering. In particular, we show that, when periodic poling is applied to just a few waveguides in the array, it creates a nonlinear defect[6,7] that possesses specific resonant scattering properties and may be employed for experimental observation of Fano resonance in nonlinear optics.

Following the waveguide design recently implemented in the study reported in Ref. 5 for observation of discrete quadratic solitons, we consider a discrete model describing an array of weakly coupled linear waveguides in which one or several neighboring waveguides have periodic poling and therefore possess a quadratic nonlinear response (see Fig. 1). When the matching conditions are satisfied, the fundamental-frequency (FF) mode with frequency $\omega$ can generate parametrically a second-harmonic (SH) wave at frequency $2\omega$, so such a structure with several poled waveguides may behave as a nonlinear defect with localized quadratic nonlinearity. The continuous version of this model was studied earlier.[8]

In the tight-binding approximation usually employed in the theory of discrete lattices,[3] the effective equations for the complex envelopes of the FF wave ($u_n$) and its SH component ($v_n$) coupled at the defect waveguides with $n=0,\ldots,N$ can be written in dimensionless form:

$$i\frac{\mathrm{d}u_n}{\mathrm{d}z}+c_u(u_{n+1}+u_{n-1})+2\sum_{m=0}^{N}u_m{}^{*}v_m\delta_{nm} = 0,$$

$$i\frac{\mathrm{d}v_n}{\mathrm{d}z}+c_v(v_{n+1}+v_{n-1})-\Delta v_n+\sum_{m=0}^{N}u_m{}^{2}\delta_{nm} = 0, \quad (1)$$

where $c_u$ and $c_v$ are the coupling coefficients, $\delta_{nm}$ is the Kronecker symbol, and $\Delta$ is a phase-mismatch parameter assumed to be identical for all waveguides.

First we analyze the scattering of a plane FF wave at frequency $\omega$ by a single ($n=0$) quadratic defect waveguide (impurity site). After interaction with the quadratic waveguide, the FF wave generates a SH wave that can either propagate or be trapped and that is guided by the defect waveguide. To calculate transmission coefficient $t(k)$ of the FF wave, we present the fields in the form

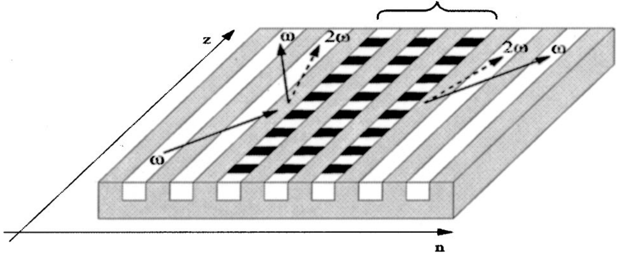

Fig. 1. Schematic of an array of channel waveguides with quadratic nonlinearity in which nonlinear defects are created by periodic poling. The arrows show the scattering process.

$$u_n(z) = \exp(i\beta_1 z)\begin{cases} I\exp(ikn)+R\exp(-ikn) & n<0 \\ T\exp(ikn) & n\geqslant 0 \end{cases}, \tag{2}$$

$$v_n(z) = \exp(i\beta_2 z)\begin{cases} \tilde{R}\exp(-iqn) & n<0 \\ \tilde{T}\exp(iqn) & n\geqslant 0 \end{cases}, \tag{3}$$

where $\beta_1 = 2c_u\cos k$ and $\beta_2 = 2c_v\cos q - \Delta$ are propagation constants of the FF and SH, respectively, and $k$ and $q$ are corresponding transverse wave numbers. Using the phase-matching condition,[8] $(2\beta_1 = \beta_2)$ we obtain the relation $c_v\cos q - (\Delta/2) = 2c_u\cos k$, which defines the dependence $q = q(k)$. For $k_{\min} < k < k_{\max}$, function $q(k)$ takes real and positive values. Outside this interval, the values of $q(k)$ are purely imaginary, and they correspond to localized (nonradiating) states trapped by the defect waveguide at $n=0$. A simple calculation yields the result $k_{\max,\min} = \cos^{-1}[(c_v \pm \Delta/2)/2c_u]$ when these values are real and positive or zero otherwise.

Evaluating the mode coupling at the impurity $(n=0)$ and neighboring $(n=-1)$ sites allows us to obtain the relations $T = I + R$ and $\tilde{R} = \tilde{T}$ and derive a nonlinear equation for the transmission coefficient, $t = |T|^2/|I|^2$, in the form $t[1+b(k)t]^2 = 1$, where $b(k) = |I|^2[2c_uc_v\sin k\sin q(k)]^{-1}$, which has only one real solution. The resonant scattering when a localized SH field is generated can be analyzed similarly by replacing $q \to iq$.

The study of wave scattering in this system predicts resonant suppression of transmission at some points, i.e., $t(k_{\min,\max}) = 0$ (see Fig. 2). We demonstrate below that these resonant reflections correspond to a novel type of the well-known Fano resonance.[9] Indeed, according to Fano theory,[9] destructive interference and resonant suppression of transmission are observed when there exists a localized state coupled to the propagating channel with the energy inside the linear spectrum. Note that $q(k_{\min}) = 0$ and $q(k_{\max}) = \pi$; i.e., these values define the band edges of the propagation spectrum of the propagating SH field, and the resonances take place when the SH field is generated. This situation seems to contradict the classical definition of Fano resonance. However, below we demonstrate in more detail that this kind of resonant scattering can be indeed defined as being associated with Fano resonance.

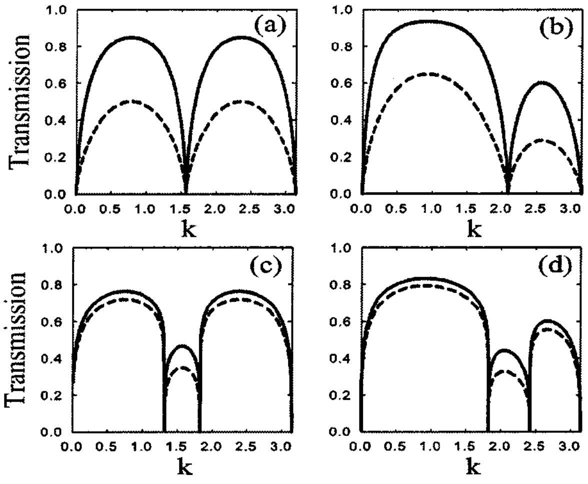

Fig. 2. Examples of the FF transmission coefficient of scattering by the quadratic impurity waveguide for $c_u = 1$ and (a) $c_v = 0$, $\Delta = 0$; (b) $c_v = 0$, $\Delta = 2$; (c) $c_v = 0.5$, $\Delta = 0$; (d) $c_v = 0.5$, $\Delta = 2$ for $I = 1$ (solid curves) and $I = 2$ (dashed curves).

First we consider the simplest case when the coupling between the SH modes of different waveguides vanishes, i.e., $c_v = 0$, which was indeed the case in recent experiments.[5] Then, in the stationary case, coupled equations (1) can be written in the form

$$\begin{aligned} \beta_1 u_n &= c_u(u_{n+1}+u_{n-1}) + 2u_0^{*} v\,\delta_{n0}, \\ 2\beta_1 v &= -\Delta v + u_0^{2}, \end{aligned} \tag{4}$$

Model (4) describes the main propagation channel for field $u_n$ and an additional discrete mode $v = v_0$ that is coupled to it parametrically; this model is similar to the so-called Fano–Anderson model,[10] except that the coupling here is nonlinear, which makes the scattering problem nonlinear.

To simplify the analysis we eliminate the discrete mode described by the second line of Eqs. (4) and obtain

$$\beta_1 u_n = c_u(u_{n+1}+u_{n-1}) + \frac{2|u_0|^2 u_0}{2\beta_1 + \Delta}\delta_{n0}, \tag{5}$$

which is an effective equation for a propagation channel that contains a scattering potential. The strength of this nonlinear resonant scattering potential depends on incoming intensity $|I|^2$ and propagation constant $\beta_1(k)$. If $\Delta$ is chosen such that $k_F$ is between 0 and $\pi$, and

$$2\beta_1(k_F) = -\Delta, \tag{6}$$

our potential becomes infinitely large for a particular frequency $\beta_1(k_F)$, which will lead to the perfect reflection. Note here that $k_F = k_{\min,\max}$ for our case $c_v = 0$. Indeed, after some algebra we can write the equation for the transmission coefficient in the form

$$t^3 + \gamma(k)(t-1) = 0, \tag{7}$$

where $\gamma(k) = (4c_u\cos k + \Delta)^2 c_u^2 \sin^2 k/|I|^4$. From Eq. (7) one can see that, when $\gamma(k) = 0$, transmission coefficient $t$ vanishes. This happens at wave numbers $k=0$ and $k=\pi$, which correspond to the band edges of the propagation spectrum, and also at $k = k_F$. In the latter case this is exactly the Fano resonance.

When the coupling between the SH modes in the waveguide array does not vanish (i.e., $c_v \neq 0$), it leads to the appearance of the spectrum of propagating SH modes, $\beta_2[q(k)]$. At the band edges of this spectrum, $\beta_2(0)$ and $\beta_2(\pi)$, which correspond to $k = k_{\min}$ and $k = k_{\max}$, the propagating SH field is described as a standing constant-amplitude mode of forms $v_n = v_0$ and $v_n = (-1)^n v_0$, where $v_0$ is constant. Therefore, for these two cases we can again obtain a single-site equation for the second scattering channel. By applying a similar approach to these particular cases, we obtain two conditions for the Fano resonance:

$$2\beta_1(k_{F_{1,2}}) = -\Delta \pm 2c_v, \tag{8}$$

which occur exactly when propagation constant $2\beta$ of the generated SH field coincides with either propagation constant $\beta_2(0)$ or $\beta_2(\pi)$, i.e., at the band edges of the linear spectrum of the propagating SH modes. In other words, in such situations we excite constant modes by a local perturbation. Since the group velocity of these modes vanishes at the band edges, any local excitation cannot propagate at the given frequency. This makes these modes effectively local and leads, finally, to the phenomenon of Fano resonance.

We note here that, in our physical system of quadratic waveguides, the coupling between the first and second propagation channels is nonlinear and therefore depends on the intensity of the incoming wave. According to Eqs. (6) and (8) the position of the Fano resonances does not depend on the value of this coupling because of its local nature.[11] As a consequence, this novel type of Fano resonance should exist for any intensity of incoming waves similar to the conventional Fano resonance in linear theory. But the width of the resonance depends on this coupling[12] and therefore on the incoming intensity of light (see Fig. 2).

To check the validity of our plane-wave analysis and the manifestation of the effect in a realistic experiment, we performed a numerical simulation of Gaussian beam scattering. The results are summarized in Fig. 3, and they are in a good agreement with the theory of plane-wave scattering.

In the case of $N$ defects and vanishing coupling between them ($c_v=0$), Fano resonance does not change its position [Eq. (6)].[13] After scattering by the first defect close to Fano resonance, other waveguides become almost transparent because of a small incoming intensity. Therefore the width of the resonance will remain almost the same as in the case of a single defect.

In conclusion, we have analyzed a novel waveguide structure in which an optical analog of the Fano resonance can be observed as peculiarities of resonant scattering and SH generation in the quadratic waveguide arrays. We believe that this kind of waveguide structure, which has already been fabricated for observation of discrete quadratic solitons,[5] is a good candidate for experimental observation of Fano resonances in nonlinear optics.

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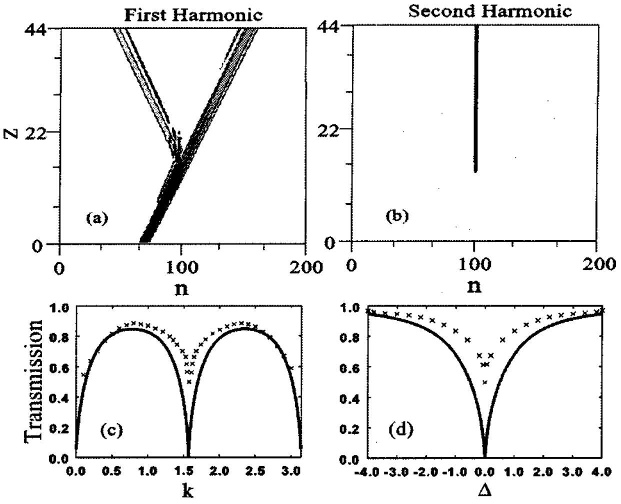

Fig. 3. Top, resonant generation of the trapped SH field by a Gaussian beam of the (a) FF and (b) SH fields for parameters $c_u=1$, $c_v=0$, $\Delta=0$, and $k=\pi/2$. Bottom, comparison of the transmission coefficients of the plane waves (solid curves) and Gaussian beam (crosses) for $c_u=1$, $c_v=0$ and (c) $\Delta=0$ and (d) $k=\pi/2$. Because of its finite spectral width, the transmission coefficient of the Gaussian beam does not vanish.

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