# Stable soliton in the fiber-optic system with self-frequency shift

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We show that when narrowband filtering and nonlinear gain are considered, long-distance propagation of stable pulses in a fiber-optic system can be realized in the presence of self-frequency shift. We analyze the system by using the perturbative approach and obtain steady-state solutions and their existence conditions. The steady-state solutions are studied by linear stability analysis. Typical examples are given for stable pulse propagation over long distances, in fiber-based transmission lines. The results are verified by direct solution of partial differential equations numerically. Finally we show that our result is also suitable for investigation of the stable soliton in fiber lasers. A typical numerical example is given. 

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## 1. INTRODUCTION

The optical soliton has been an active topic of study during the past three decades because of its special characteristics and potential applications.[1,2] For example it can be applied in ultrahigh-speed optical telecommunication systems, since the pulse shape of a soliton can be preserved over an indefinitely long distance by balancing the effect of anomalous group-velocity dispersion (GVD) with the nonlinear effect of self-phase modulation.[3] Propagation of picosecond optical solitary pulses is governed by the nonlinear Schrödinger (NLS) equation. The NLS equation is completely integrable and its solutions can be obtained by different methods. For much shorter pulses, such as subpicosecond or femtosecond optical pulses, the higher-order effects such as third-order dispersion, self-steepening, and self-frequency shift (SFS) become important. To take account of all these higher-order effects the NLS equation has to be modified as a higher-order nonlinear Schrödinger equation.[4] Only a few kinds of these equations are completely integrable by inverse-scatteringlike methods.[5–6]

In the absence of the SFS effect, a number of soliton (or solitary wave) solutions have been found under the balance of the self-phase modulation, GVD, third-order dispersion, and self-steepening effects in recent years.[7–16] However when the SFS effect is considered, to our knowledge analytic solutions for higher-order nonlinear Schrödinger equations have not yet been found. The soliton SFS was first observed in experiments in which solitons of a few hundred femtoseconds propagated down a fiber.[17] It was also observed in an experiment in which a high-powered, ~2-ps pulse from a dye laser was launched into a fiber.[18] The SFS is a self-induced redshift in the pulse spectrum arising from stimulated Raman scattering.[17,19] The long-wavelength components of the pulse experience Raman gain at the expense of the short-wavelength components, resulting in an increasing redshift as the pulse propagates. It has been recognized that the SFS is a potentially detrimental effect in soliton communication systems.[20] To maintain the propagation of the soliton under the influence of SFS, Blow *et al.* employed bandwidth-limited amplification to suppress the SFS through use of perturbation theory and numerical simulation.[21,22] But as mentioned by them, for long distances their method would cease to be accurate. Later Nakazawa *et al.* proposed a new technique for achieving subpicosecond–femtosecond optical soliton communication over long distances.[23,24] They also pointed out that the soliton SFS could be completely compensated for by the bandwidth-limited optical gain. Recently Ding and Kikuchi have used distributed-optical-fiber amplifiers with frequency-dependent gain to compensate for the soliton SFS.[25] By numerical simulations they have shown that low-loss, dispersion-shifted, distributed-$Er^{3+}$-doped fiber can provide almost distortion-free soliton SFS compensation for 810-fs pulses over 30 km. However in these cases some excess gain must be provided, leading to instability of the background and limiting greatly the transmission distance. Moreover the effects of SFS in both ultrashort-pulse laser and amplifier systems have been reported in recent years, although all of these systems included bandwidth-limited gain media.[26,27] Therefore to realize stable optical pulse propagation in a system with SFS, further research is needed. Recently nonlinear gain has been introduced to suppress linear-wave growth.[28]

In this paper we concentrate on the system with SFS effect. Narrowband filtering is added to the system to reduce the frequency shift and nonlinear gain is used to suppress instability of the background. We show that when narrowband filtering and nonlinear gain are considered simultaneously, stable optical pulse propagation over long distances can be realized in fiber-based transmission systems with SFS. In addition we show that our result is suitable for investigation of the stable soliton in fiber lasers.

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The paper is organized as follows. In Section 2 we briefly introduce the model and present steady-state solutions and their existence conditions. In Section 3 the steady-state solutions are investigated by linear stability analysis. In Section 4 typical numerical examples are given for fiber-based transmission line and fiber lasers. We conclude the paper in Section 5.

## 2. MODEL AND STEADY-STATE SOLUTIONS

The propagation of picosecond pulses in a lossless optical fiber can be described by the standard NLS equation[29]

$$\frac{\partial U}{\partial z} = i\left[\frac{1}{2}\frac{\partial^2 U}{\partial t^2} + |U|^2 U\right], \tag{1}$$

which includes the effects of GVD and the Kerr nonlinear refractive index, namely, self-phase modulation. Here $z$ is the normalized propagation distance, $t$ is the retarded time, and $U$ is the complex envelope of the electric field.

It is well known that the NLS equation [Eq. (1)] is completely integrable and supports bright solitons in the anomalous dispersion regime and dark solitons in the normal dispersion regime. The lowest-order bright soliton pulse is quite useful for high-speed, long-distance communication systems because it does not change its waveform during propagation along an optical fiber. The solution can be written in the form[29]

$$U(z, t) = \frac{1}{\tau}\exp\left\{\frac{iz}{2\tau^2}\right\}\operatorname{sech}\left\{\frac{t}{\tau}\right\}. \tag{2}$$

When the SFS, the narrowband filter, and the nonlinear gain are considered Eq. (1) should be modified as

$$\frac{\partial U}{\partial z} = i\left[\frac{1}{2}\frac{\partial^2 U}{\partial t^2} + |U|^2 U\right] + i t_d U\frac{\partial |U|^2}{\partial t} + a_0 U + a_1\frac{\partial^2 U}{\partial t^2} + a_2|U|^2U + a_3|U|^4U, \tag{3}$$

where $t_d$ results from the time-retarded, induced-Raman process and is responsible for the soliton SFS, $a_0 > 0$ ($<0$) is the linear excess gain (loss), $a_1$ describes the effect of spectral limitation which is due to the bandwidth-limited filter, and $a_2$ and $a_3$ account for nonlinear gains.

Unfortunately there is no exact analytical soliton solution for Eq. (3). To study the one-soliton dynamic characteristics in such a system, we may analyze them by evaluating the evolution of the soliton parameters. That is, based on the one-soliton solution of Eq. (2), we assume the soliton parameters are slowly varying functions of the distance $z$. In addition considering the possible shift of the center frequency because of stimulated Raman scattering, we take the generalized trying function as[21]

$$U(z, t) = \frac{1}{\tau(z)}\operatorname{sech}\left[\frac{t-\nu(z)}{\tau(z)}\right] \times \exp(-i\{\omega(z)[t-\nu(z)] - \alpha(z)\}). \tag{4}$$

In Eq. (4) there are four unknown parameters that evolve slowly along the distance $z$: the pulse duration $\tau(z)$, the frequency shift $\omega(z)$, the pulse peak position $\nu(z)$, and the phase shift $\alpha(z)$. To understand the evolution of these parameters, we can derive the equations for them based on the conserved quantities of the NLS equation, the lowest two of which are energy and momentum. For the perturbed NLS system the energy and momentum are not conserved and evolve slowly with distance. Multiplying Eq. (3) by $U^*(z, t)$ and by $i\partial U^*/\partial t$, then integrating them over time from $-\infty$ to $+\infty$, we can easily obtain two complex equations for the integrated quantities. The imaginary parts of these two resulting equations describe the evolution of energy $\int|U|^2$ and momentum $\int i(U\partial^*U/\partial t - U^*\partial U/\partial t)$. The real parts will represent the average phase and group velocities. By substituting Eq. (4) into these two complex equations for the integrated quantities, we obtain the evolution equations

$$\frac{d\tau}{dz} = 2\left(a_1\omega^2\tau + \frac{a_1}{3\tau} - a_0\tau\right) - \frac{4a_2}{3\tau} - \frac{16a_3}{15\tau^3}, \tag{5a}$$

$$\frac{d\omega}{dz} = \frac{8t_d}{15\tau^4} - \frac{4a_1\omega}{3\tau^2}, \tag{5b}$$

for the pulse duration $\tau(z)$ and frequency shift $\omega(z)$, respectively; and the equations

$$\frac{d\alpha}{dz} = -\frac{1}{2}\omega^2 - \frac{1}{2\tau^2}, \tag{5c}$$

$$\frac{d\nu}{dz} = -\omega, \tag{5d}$$

for phase shift $\alpha(z)$ and peak position $\nu(z)$, respectively, which are dependent on $\tau$ and $\omega$.

When we consider only the SFS effect and delete the terms dependent on gain (i.e., $a_0 = a_1 = a_2 = a_3 = 0$) from Eqs. (5a) and (5b), one clearly finds that the pulse duration $\tau$ does not change but the frequency shift $\omega$ increases linearly with the distance $z$, as reported in Refs. 21 and 29. It is easily proven that these equations are in agreement with those obtained by other perturbation methods (e.g., Lagrangian method[30,31]).

When the bandwidth-limited gain and nonlinear gain terms are taken into account, Eqs. (5a) and (5b) have the steady-state solutions $\tau = \tau_f$ and $\omega = \omega_f$, where $\tau_f$ and $\omega_f$ respectively satisfy

$$\tau_f^4 - \frac{a_1 - 2a_2}{3a_0}\tau_f^2 + \frac{40a_1a_3 - 12t_d^2}{75a_0a_1} = 0, \tag{6a}$$

$$\omega_f = \frac{2t_d}{5a_1\tau_f^2}. \tag{6b}$$

Since $\tau$ is the pulse width parameter, we must require $\tau_f$ to be positive. Under this condition, from Eq. (6a) we find that there may exist one or two steady-state solutions, depending on the parameter values of the additional terms. In specific terms,

1. if $(10a_1a_3 - 3t_d^2)/(a_0a_1) < 0$, there is one steady-state solution.
2. if $(a_1 - 2a_2)/(3a_0) > 0.8[(10a_1a_3 - 3t_d^2)/(3a_0a_1)]^{1/2} > 0$, there are two steady-state solutions.

In addition, there exists one steady-state solution in the following three critical cases:

3. $(a_1 - 2a_2)/(3a_0) = 0.8[(10a_1a_3 - 3t_d^2)/(3a_0a_1)]^{1/2} > 0$.
4. $a_1 = 2a_2$ and $(10a_1a_3 - 3t_d^2)/(a_0a_1) < 0$.
5. $10a_1a_3 = 3t_d^2$ and $(a_1 - 2a_2)/a_0 > 0$.

It should be noted that, in the absence of nonlinear gain terms ($a_2 = a_3 = 0$), there exists one steady-state solution corresponding to case 1. In this case some excess gain ($a_0 > 0$) must be provided so that bandwidth-limited gain can compensate for the SFS to form the steady-state solution. However the excess gain will amplify linear waves (noise) coexistent with the soliton pulses, leading to instability of the background and eventually to the breakup of the soliton pulses.[21] In contrast the existence of nonlinear gain terms provides two other possibilities (cases 2 and 3) to form steady-state solutions. In what follows we will examine the stability of these steady-state solutions.

## 3. LINEAR STABILITY ANALYSIS OF THE STEADY-STATE SOLUTION

Setting $\tau = \tau_f + \Delta\tau$ and $\omega = \omega_f + \Delta\omega$ and linearizing Eqs. (5a) and (5b) around the steady-state solution $(\tau_f, \omega_f)$, we derive the evolution equations for the small deviations ($\Delta\tau$ and $\Delta\omega$) in the matrix form

$$\begin{pmatrix} \dfrac{d\Delta\tau}{dz} \\ \dfrac{d\Delta\omega}{dz} \end{pmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} \Delta\tau \\ \Delta\omega \end{pmatrix}, \tag{7}$$

where

$$C_{11} = 2a_1\omega_f^2 - 2a_0 + \frac{4a_2 - 2a_1}{3\tau_f^2} + \frac{16a_3}{5\tau_f^4},$$

$$C_{12} = 4a_1\omega_f\tau_f,$$

$$C_{21} = \frac{8a_1\omega_f}{3\tau_f^3} - \frac{32t_d}{15\tau_f^5},$$

$$C_{22} = -\frac{4a_1}{3\tau_f^2}.$$

The two eigenvalues $(\lambda_1, \lambda_2)$ of the $2 \times 2$ matrix of Eq. (7) satisfy

$$\lambda^2 - (C_{11} + C_{22})\lambda + C_{11}C_{22} - C_{21}C_{12} = 0. \tag{8}$$

After some algebraic calculations, we can obtain

$$\lambda^2 - \frac{8}{15\tau_f^4}[8a_3 + 5(a_2 - a_1)\tau_f^2]\lambda + \frac{16a_1}{9\tau_f^4}[6a_0\tau_f^2 - a_1 + 2a_2] = 0. \tag{9}$$

We know that the steady-state solution $(\tau_f, \omega_f)$ is linearly stable if both the real parts of the two eigenvalues $(\lambda_1, \lambda_2)$ are negative. Considering that the parameter $a_1$ is generally positive, from this eigenvalue equation we find that the stable steady-state solution $(\tau_f, \omega_f)$ can be realized if the conditions

$$8a_3 + 5(a_2 - a_1)\tau_f^2 < 0, \tag{10}$$

$$6a_0\tau_f^2 - a_1 + 2a_2 > 0, \tag{11}$$

are satisfied.

Further, combining the existence conditions of the steady-state solution prescribed in Section 2, we find that only if the condition $a_2 > 0.5(a_1 - 6a_0\tau_f^2)$ and simultaneously the condition

$$\frac{3t_d^2}{10a_1} \leqslant a_3 < \frac{5}{8}(a_1 - a_2)\tau_f^2, \tag{12}$$

or

$$\left[\frac{3t_d^2}{10a_1} + \frac{5(a_1 - 2a_2)^2}{96a_0}\right] \leqslant a_3 < \min\left[\frac{3t_d^2}{10a_1}, \frac{5}{8}(a_1 - a_2)\tau_f^2\right], \tag{13}$$

are satisfied does the system admit of steady-state solutions that are linearly stable. Furthermore if $a_0 < 0$ is imposed on the conditions, not only the steady-state solutions but the background are stable.

## 4. NUMERICAL EXAMPLES

Now we give an example of stable pulse propagation over long distances. In a real system we assume that the filters are periodically inserted in the transmission line at a rate of M filters per unit of distance and the amplifier is modeled by complex Lorentzian line shape.[32] We stipulate that one dispersion length corresponds to one unit of distance. The dispersion length is defined as $L_D = T_0^2/|\beta|$ where $T_0$ is the pulse-width parameter and $\beta$ is the GVD parameter. By setting other parameters as $1.763T_0 = 400$ fs, $\lambda = 1.55\ \mu$m, filter spacing $Z_a = 1/M = 0.25$ ($M = 4$), GVD value of dispersion shifted fiber $D = -0.5$ ps/(nm·km), nonlinear parameter $\gamma = 20\ \text{W}^{-1}\,\text{km}^{-1}$, fiber loss $\alpha = 0.25$ dB/km, optical gain $g_0 = 0.25$ dB/km, loss of each filter 0.05 dB, 3-dB bandwidth of the filter is $\Delta\lambda = \lambda^2/[\pi c T_0(2a_1Z_a)^{1/2}] = 29$ nm, nonlinear gain $g_2 = 6.8\ \text{W}^{-1}\,\text{km}^{-1}$, and slope of Raman gain is $5.9 \times 10^{-15}$ s, we can get that $L_D = 81.6$ m, $t_d = -0.05$, and the system parameters are $a_0 = -0.05$, $a_1 = 0.3$, $a_2 = 0.5$, and $a_3 = -0.34$, which are the same as those used in Ref. 28.

By calculation we find that these parameter values satisfy case 2 in Section 2, that is, Eqs. (5a) and (5b) have two steady-state solutions, the first $\tau_{f1} \approx 0.9975$, $\omega_{f1} \approx -0.0670$; and the second $\tau_{f2} \approx 1.9162$, $\omega_{f2} \approx -0.0182$. By use of Eq. (9) we can get the eigenvalues for the two solutions as $\lambda_1 \approx -0.4646 - 0.2855i$, $\lambda_2 \approx -0.4646 + 0.2855i$; and $\lambda_1 \approx -0.1124$, $\lambda_2 \approx 0.1501$, respectively. From these values we can infer that the first solution is linearly stable and the second one is linearly unstable.

The results are proved by the numerical results of Runge–Kutta integration of nonlinear ordinary differen-

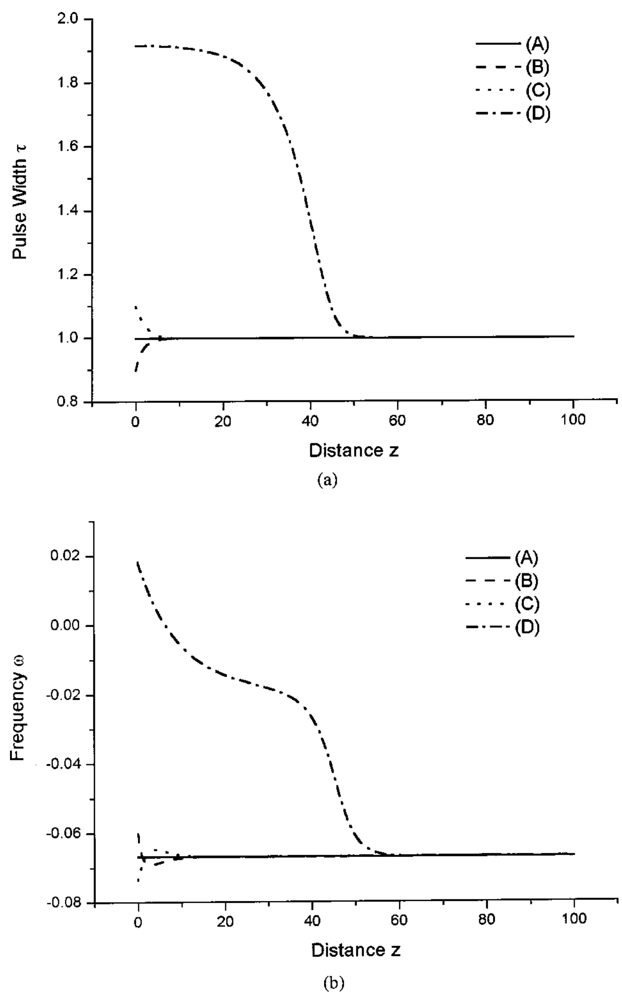


Fig. 1. (a) Pulse temporal width and (b) frequency shift versus the normalized distance $z$ with different initial conditions. The parameters are $t_d = -0.05$, $a_0 = -0.05$, $a_1 = 0.3$, $a_2 = 0.5$, and $a_3 = -0.34$.

tial Eqs. (5a) and (5b). Figures 1(a) and 1(b) show the evolution of temporal width $\tau$ and frequency $\omega$, respectively, of the pulse to a distance of $z = 100$. Curve (A) corresponds to the first steady-state solution, that is, $[\tau(0),\ \omega(0)] = (\tau_{f1},\ \omega_{f1})$. It can be clearly seen that the two values remain constant. When the initial steady-state solution is perturbed, for example by increasing or decreasing the exact values by 10%, i.e., $(0.9\tau_{f1},\ 0.9\omega_{f1})$ or $(1.1\tau_{f1},\ 1.1\omega_{f1})$—by the same method as that mentioned in Ref. 33—the evolution results are shown in Fig. 1 by curve (B) and curve (C). We can see that both the pulse temporal width and frequency are attracted to the fixed values of the solution $\tau_{f1} \approx 0.9975$, $\omega_{f1} \approx -0.0670$ after a short period of adjustment. In contrast, as shown by curve (D), the second steady-state solution $[\tau(0),\ \omega(0)] = (\tau_{f2},\ \omega_{f2})$ is attracted to the first one. This indicates that the first steady-state solution is an attractor. These results agree well with those from the linear stability analysis.

Next we show the evolution of the optical pulse with initial form $U(0, t) = \mathrm{Sech}(t)$ by directly solving partial differential Eq. (3) with split-step Fourier transformation. This initial pulse can be obtained by setting $\tau = 1$ and $\omega = 0$ in Eq. (4), which approximately corresponds to the first steady-state solution $\tau_{f1} \approx 0.9975$, $\omega_{f1} \approx -0.0670$. Figure 2(a) shows the evolution plot of the pulse to a normalized distance of $z = 100$. We can clearly see that the optical pulse becomes stable after a short period of adjustment and propagates without distortion. In Fig. 2(b) we show the contour of the frequency spectrum of the pulse corresponding to that in Fig. 2(a). From it we can barely detect the frequency shift after the stable pulse is achieved. This implies that the SFS effect has been greatly suppressed. In fact the stable optical pulse can propagate even farther. Our new numerical results have shown that the optical pulse retains its shape even to a distance of $z = 13{,}000$, which corresponds to 1060.8 km, and there is barely any change in the spectrum after the stable pulse is formed. We may therefore infer that stable pulse transmission over long distances can be realized in a practical system if we employ narrowband filtering and nonlinear gain to compensate for SFS and reduce the instability of the background appropriately.

After many calculations we found that only the optical pulse with narrower duration is stable if there exist two steady-state solutions. We did not observe that the two steady-state solutions are stable simultaneously. By changing the values of the filter and nonlinear gain parameters we found that the pulse width of a stable pulse is broadened if the filter effect is strengthened. On the other hand, if the nonlinear gain is increased the pulse duration is narrowed. Similar results have been reported for passively mode-locked fiber lasers.[34]

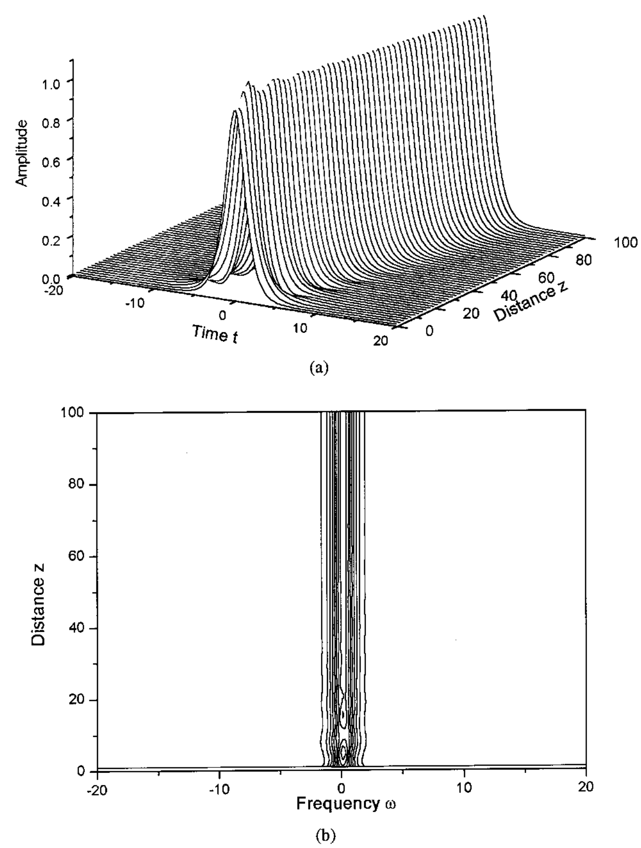


Fig. 2. (a) Evolution of the stable pulse obtained by direct numerical simulation; (b) contour plot of the frequency spectrum for the case of (a). The parameters are the same as those in Fig. 1.

Finally, since our model is also adapted to the investigation of the stable soliton in fiber lasers, we give here an example for this case. The parameters of Eq. (3) are set as $a_0 = 0.01$, $a_1 = 0.15$, $a_2 = 0.06$, $a_3 = 0$. These parameters are typical of fiber lasers and their significance can be easily found in Ref. 35. Our calculations showed that these parameters satisfied case 1. in Section 2; that is, Eqs. (5a) and (5b) have one steady-state solution $\tau_f \approx 1.1040$, $\omega_f \approx -0.1094$. The solution is stable since the eigenvalues are $\lambda_1 \approx -0.1554$, $\lambda_2 \approx -0.0415$. The numerical results obtained by Runge–Kutta integration of Eqs. (5a) and (5b) are shown in Fig. 3. Figures 3(a) and 3(b) show the evolution of temporal width and frequency of the pulse to a distance of 100. Curves, (A), (B), and (C) are the evolution results when the initial values are given as $\tau_f$, $\omega_f$; $0.9\tau_f$, $0.9\omega_f$; and $1.1\tau_f$, $1.1\omega_f$, respectively. The results show that the solution is stable against small deviations, which agrees well with the theoretical results.

The numerical evolution results of the partial differential Eq. (3) solved by direct use of the split-step Fourier method are shown in Fig. 4. Figures 4(a) and 4(b) show the evolution of the pulse and the corresponding spectrum, respectively. Obviously the directly integrated results also show that a stable soliton can be obtained in such a system.

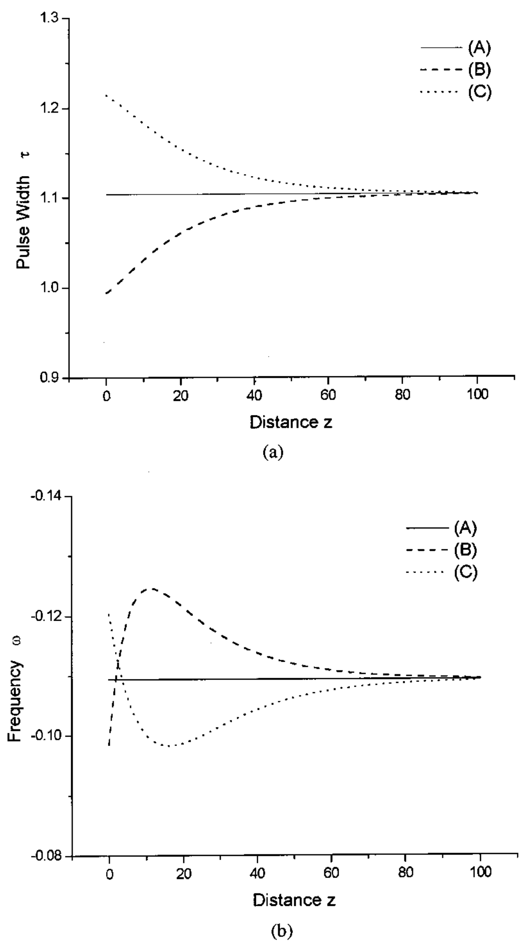


Fig. 3. (a) Pulse temporal width and (b) frequency shift versus the normalized distance $z$ with different initial conditions. The parameters are $t_d = -0.05$, $a_0 = 0.01$, $a_1 = 0.15$, $a_2 = 0.06$, and $a_3 = 0$.

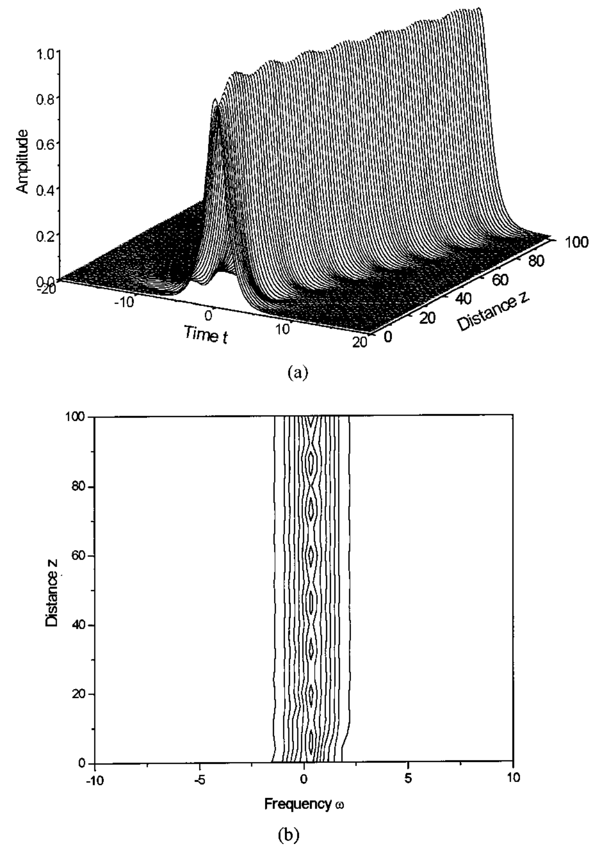


Fig. 4. (a) Evolution of the stable pulse obtained by direct numerical simulation; (b) contour plot of the frequency spectrum for the case of (a). The parameters are the same as those in Fig. 3.

We would like to point out that in a fiber laser system, when $a_0 > 0$ (it should not be too large), a stable pulse can be obtained, but in long-distance transmission lines, $a_0$ should be smaller than zero. This is because even small linear gain will amplify the dispersion wave, lead to instability of the background, and destroy the shape of the pulse after propagating a long distance.

## 5. CONCLUSION

We have considered the propagation of pulses in fiber-optic systems with self-frequency shift. Narrowband filtering is added to the system to reduce the frequency shift. To avoid instability of the background, we use nonlinear gain in the system to compensate for the loss resulting from the filter and the optical fiber. We show that when narrowband filtering and nonlinear gain are considered, stable optical pulse propagation over long distances can be realized in fiber-based transmission systems with self-frequency shift. We analyze the system by using a perturbative approach and obtain steady-state solutions and their existence conditions. The steady-state solutions are studied by linear stability analysis. Typical examples are given for stable pulse propagation over long distances in fiber-based transmission lines. The results are verified by directly solving partial differential equations numerically. Finally, we show that our result is also suitable for investigation of the stable soliton in fiber lasers. A typical numerical example is given.

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