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SOLITON PROPAGATION IN NONUNIFORM OPTICAL FIBERS

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In this paper, we construct the Lax pair for a soliton transmission system in nonuniform optical fibers and give N-soliton solution using the Darboux transformation. The explicit one-soliton and two-soliton solutions are presented. Further, we discuss the interaction scenario between two neighboring solitons and the effect of the inhomogeneities of the fiber (z_0) on the interaction between two neighboring solitons.

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1. Introduction

It is well known that the propagation of ultrashort optical pulses in optical fibers has been the object of extensive theoretical and experimental studies in nonlinear science during the last three decades. Theoretically, a central and very important topic in the field is the study of integrable systems. In recent years, Hirota direct method, inverse-scattering transform, Painlevé property, Ablowitz–Kaup–Newell–Segur (AKNS) method, exact solutions and other problems associated with nonlinear Schrödinger (NLS) equation and higher-order nonlinear Schrödinger (HNLS) equation have been investigated in detail. But of particular interest is how to find the new solutions of the equations.

Nonlinear pulse propagation in a long-distance, high-speed optical fiber transmission system can be described by the (perturbed) nonlinear Schrödinger (NLS) equation. The NLS equation includes the linear effect due to the group velocity of the pulse and the nonlinear effect due to the Kerr effect.[1] Many research works on the development of such a system have been concentrated efforts on overcoming or controlling these effects.[2,3] However, in a real fiber, the core medium is not homogeneous.[4,5] There will always be some nonuniformity due to many factors and important among them are: (i) that which arises from a variation in the lattice parameters of the fiber medium, so that the distance between two neighboring atoms is not constant through the fiber, (ii) that due to the variation of the fiber geometry (diameter fluctuations, etc). These nonuniformities influence various effects such as loss (or gain), dispersion, phase modulation, etc.[6] When considering the inhomogeneities in the fiber, the dynamics of the optical pulse propagation is governed by the equation

$$iq_z + \alpha_1 q_{tt} + \alpha_2 |q|^2 q + iF(z)q = 0 \tag{1}$$

where $F(z)$ is the inhomogeneous parameter related to gain (or loss). Here we take

$$F(z) = \frac{1}{2(z+z_0)}\,. \tag{2}$$

Then Eq. (1) becomes

$$iq_z + \alpha_1 q_{tt} + \alpha_2 |q|^2 q + \frac{i}{2(z+z_0)} q = 0\,, \tag{3}$$

where we take $\alpha_1 = \frac{1}{2}$, $\alpha_2 = \mu^2$. Equation (3) was studied by Burtsev *et al.*[7] from the soliton point of view. In that, they have presented the Lax pair for the system equation (3) with a variable spectral eigenvalue parameter (i.e. an eigenvalue parameter as a function of either time or (and) space). Bäcklund transformation related to Eq. (3) has been presented in Ref. 8 where only one-soliton solution was given.

In this paper, we construct the Lax pair for a soliton transmission system in nonuniform optical fibers. Based on the Lax pair, N-soliton solution is presented by employing simple, straightforward Darboux transformation. And the explicit one-soliton and two-soliton solutions are presented and their properties are also analyzed. In addition, we discuss the interaction scenario between two neighboring solitons.

2. Darboux Transformation and Exact Solution

Consider the following spectral problem

$$\psi_t = U\psi\,, \tag{4}$$

$$\psi_z = V\psi\,, \tag{5}$$

where

$$U = \lambda J + P\,,$$

$$V = i\lambda^2 J + i\lambda P + \frac{i}{2} W\,,$$

and

$$J = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad P = \begin{pmatrix} 0 & \mu q \\ -\mu\overline{q} & 0 \end{pmatrix}, \quad W = \begin{pmatrix} \mu^2|q|^2 & \mu q_t \\ \mu\overline{q}_t & -\mu^2|q|^2 \end{pmatrix}.$$

The compatibility condition $U_z - V_t + [U, V] = 0$ gives Eq. (3) and λ is the variable spectral parameters given by

$$\lambda = i\frac{t-\upsilon}{2(z+z_0)}, \quad \lambda_t = i\frac{1}{2(z+z_0)}, \quad \lambda_z = -\frac{\lambda}{(z+z_0)}, \quad \upsilon = \mathrm{Re}(\upsilon) + i\,\mathrm{Im}(\upsilon),$$

$$\lambda = \alpha(z,t) + i\zeta(z,t), \quad \alpha(z,t) = \frac{\mathrm{Im}(\upsilon)}{2(z+z_0)}, \quad \zeta(z,t) = \frac{t-\mathrm{Re}(\upsilon)}{2(z+z_0)}.$$

Here $\mathrm{Re}(\upsilon)$ and $\mathrm{Im}(\upsilon)$ are, respectively, the real and imaginary parts of the hidden isospectral parameter υ. The Lax pair assures the complete integrability of a nonlinear system and is specially used to obtain integrability condition and N-soliton solutions by means of inverse scattering transform method. In this paper, we investigate Eq. (3) by employing the simple, straightforward Darboux transformation.[9–11]

Introducing transformation

$$\varphi' = (\lambda I - S)\varphi, \quad S = H\Lambda H^{-1}, \quad \Lambda = \mathrm{diag}(\lambda_1, \lambda_2), \tag{6}$$

where H is a nonsingular matrix, requiring

$$\varphi'_t = U'\varphi', \quad U' = \lambda J + P', \quad P' = \begin{pmatrix} 0 & \mu q' \\ -\mu\overline{q}' & 0 \end{pmatrix}, \tag{7}$$

and combining (4), (6) and (7), we obtain the Darboux transformation for Eq. (3) in the form:

$$P' = P + JS - SJ. \tag{8}$$

It is easy to verify that, if $(\varphi_1, \varphi_2)^T$ is a solution of (4) corresponding to eigenvalue $\lambda = \lambda_1$, $(-\overline{\varphi}_2, \overline{\varphi}_1)^T$ is also a solution of (4) and the eigenvalue λ is replaced by $-\overline{\lambda}_1$. If we take

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & -\overline{\lambda}_1 \end{pmatrix}, \quad H = \begin{pmatrix} \varphi_1 & -\overline{\varphi}_2 \\ \varphi_2 & \overline{\varphi}_1 \end{pmatrix},$$

then

$$S_{ij} = -\overline{\lambda}_1\delta_{ij} + \frac{(\lambda_1 + \overline{\lambda}_1)\varphi_i\overline{\varphi}_j}{\Delta}, \quad (i,j = 1,2), \tag{9}$$

where

$$\Delta = \det|H| = |\varphi_1|^2 + |\varphi_2|^2.$$

From Eq. (8) we obtain other solutions:

$$q' = q + \frac{2}{\mu}S_{12}; \quad \overline{q}' = \overline{q} + \frac{2}{\mu}S_{21}. \tag{10}$$

Thus we obtain the fundamental expression of Darboux transformation.

Analogous to this procedure and taking the Darboux transformation n times, we find the following formula

$$Q[n] = Q + \frac{2}{\mu} \sum \frac{(\lambda_k + \overline{\lambda}_k)\psi_1[k]\overline{\psi}_2[k]}{|\prod D_m \varphi_1|^2 + |\prod D_m \varphi_2|^2}\,, \tag{11}$$

where $k = 1, \ldots, n$, $m = 1, \ldots, n$, and

$$\begin{aligned} \psi_i[k] &= \prod D_m \varphi_i\,,\ m = 1, \ldots, k\,, \\ D_m &= \lambda_n I - S^m\,, \\ S^m_{ij} &= -\overline{\lambda}_m \delta_{ij} + \frac{(\lambda_m + \overline{\lambda}_m)\psi_i[m, \lambda_m]\overline{\psi}_j[m, \lambda_m]}{(\psi[m, \lambda_m], \psi[m, \lambda_m])}\,, \\ \psi[1, \lambda] &= \varphi \end{aligned}$$

where $\psi[1, \lambda]$ is the eigenfunction corresponding to λ for φ_1 and φ_2. Substituting zero solution $q = 0$ of Eq. (3) into Eq. (11), one can simply obtain the multisoliton solution for Eq. (3).

By setting $n = 1$ in Eq. (11), one-soliton solution can be given in the following:

$$q = \frac{4\frac{\alpha}{\mu} C_1 \overline{C}_2 \exp(2i\theta)}{|C_1|^2 \exp(-2\xi) + |C_2|^2 \exp(2\xi)} \tag{12}$$

where

$$\begin{aligned} \xi &= \int 2\alpha\zeta dz = -\frac{\mathrm{Im}(\upsilon)(t - \mathrm{Re}(\upsilon))}{2(z + z_0)} + T, \qquad \xi = -\frac{\mathrm{Im}(\upsilon)(t - \mathrm{Re}(\upsilon))}{2(z + z_0)}\,, \\ \theta &= \int (\alpha^2 - \zeta^2) dz = -(\mathrm{Im}^2(\upsilon) - (t - \mathrm{Re}(\upsilon))^2)\,\frac{1}{4(z + z_0)}\,, \end{aligned}$$

and T is the integration constant. Thus we have derived the exact soliton solution for the wave propagation in the inhomogeneous optical fiber system using Darboux transformation. If we take $C_1 = C_2$, we have the one-soliton solution as follows

$$q = \frac{2\alpha}{\mu}\ \mathrm{sech}\, 2\xi\ \exp(2i\theta)\,. \tag{13}$$

Similarly, setting $n = 2$, the two-soliton solution can be written in an explicit form as follows

$$q[2] = \frac{G}{F}\,, \tag{14}$$

$$\begin{aligned} G &= [a_1(z,t) + a_3(z,t)] \cosh(2\xi_2(z,t)) e^{i2\theta_1(z,t)} \\ &\quad + [a_2(z,t) + a_4(z,t)] \cosh(2\xi_1(z,t)) e^{i2\theta_2(z,t)} \\ &\quad + a_5(z,t) \left[\sinh(2\xi_1(z,t)) e^{2i\theta_2(z,t)} - \sinh(2\xi_2(z,t)) e^{2i\theta_1(z,t)}\right], \\ F &= b_1(z,t) \cosh(2\xi^+) + b_2(z,t) \cosh(2\xi^-) + b_3(z,t) \cos(2\theta^-)\,, \end{aligned}$$

where

$$\xi^+ = \xi_1(z,t) + \xi_2(z,t)\,; \quad \xi^- = \xi_2(z,t) - \xi_1(z,t)\,,$$
$$\theta^+ = \theta_2(z,t) + \theta_1(z,t)\,; \quad \theta^- = \theta_2(z,t) - \theta_1(z,t)\,,$$

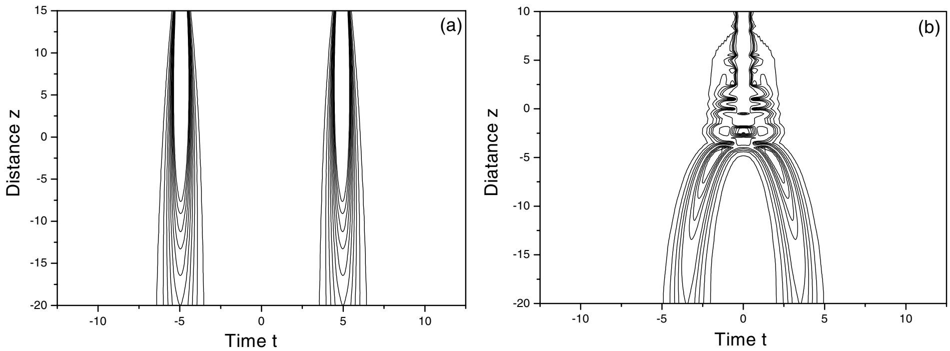
and the explicit form of $\xi_k(z,t)$ and $\theta_k(z,t)$ can be respectively derived from the following equations using the spectral parameter $\lambda(z)$.

$$\begin{aligned}
\lambda_k(z) &= \alpha_k(z) + i\zeta_k(z)\,,\\
\xi_k &= \int 2\alpha_k\zeta_k dz = -\frac{\mathrm{Im}(\upsilon_k)(t - \mathrm{Re}(\upsilon_k))}{2(z+z_0)} + T_k\,,\\
\theta_k &= \int (\alpha_k^2 - \zeta_k^2)dz\\
&= -(\mathrm{Im}^2(\upsilon_k) - (t - \mathrm{Re}(\upsilon_k))^2)\frac{1}{4(z+z_0)}\,, \quad k = 1,2\,,
\end{aligned}$$

where T_k is the integration constant and introducing notations with the spectral parameter $\lambda(z)$ as follows

$$\begin{aligned}
a_1(z,t) &= -\frac{\alpha_1(z,t)\alpha^+\alpha^-}{\mu}\,,\\
a_2(z,t) &= \frac{\alpha_2(z)\alpha^+\alpha^-}{\mu}\,,\\
a_3(z,t) &= \frac{\alpha_1(z,t)(\zeta^-)^2}{\mu}\,,\\
a_4(z,t) &= \frac{\alpha_2(z,t)(\zeta^-)^2}{\mu}\,,\\
a_5(z,t) &= \frac{2i\alpha_1(z,t)\alpha_2(z,t)\zeta^-}{\mu}\,,\\
b_1(z,t) &= \frac{(\alpha^-)^2 + (\zeta^-)^2}{4}\,,\\
b_2(z,t) &= \frac{(\alpha^+)^2 + (\zeta^-)^2}{4}\,,\\
b_3(z,t) &= -\,\alpha_1(z,t)\alpha_2(z,t)\,,\\
\alpha^+ &= \alpha_2(z,t) + \alpha_1(z,t)\,,\\
\alpha^- &= \alpha_2(z,t) - \alpha_1(z,t)\,.
\end{aligned}$$

Thus we have derived the exact two-soliton solution for the wave propagation in the inhomogeneous optical fiber system equation. These formulae will be important for the study of interaction solitons under the influence of perturbations.



(a) (b)

Fig. 1. Interaction of two equal amplitude pulses with the same compression factor $z_0 = -22.6$ and with initial pulse separation: (a) $T_0 = 10$, (b) $T_0 = 7$.

3. Scenario of Interaction Between Two Neighboring Solitons

In order to discuss the interaction between two neighboring solitons, let us consider initial pulse in the following form:

$$q = \mathrm{sech}\left(t - \frac{T_0}{2}\right) + A\,\mathrm{sech}\left(t + \frac{T_0}{2}\right) , \tag{15}$$

where T_0 is the soliton separation. First we consider the interaction scenario between two neighboring solitons with equal amplitudes in the presence of the inhomogeneities of the fiber. Figure 1(a) depicts the contour of propagation of two solitons with equal amplitude with initial pulse separation $T_0 = 10$. As shown in Fig. 1(a), we see clearly that their interaction cannot occur if the separation between two neighboring solitons is large enough. However, as the separation of two pulses decreases further, which is shown in Fig. 1(b) (here we take $T_0 = 7$), it is noted that the interaction between two neighboring solitons becomes serious. The above two cases are according to the same pulse compression factor $z_0 = -22.6$. From Fig. 1, we can see clearly that the compression of the pulse can be achieved due to the inhomogeneities of the fiber. However, as we take two pulses with unequal amplitudes, such as taking $A = 1.5$ here, we will see the scenario of interaction is different. Figure 2 depicts the interaction between two neighboring solitons with unequal amplitudes. As shown in Fig. 2, the interaction force between two neighboring solitons becomes weaker, which is very helpful for optical soliton communication.

In order to see the effect of the inhomogeneities of the fiber (z_0) on the interaction between two neighboring solitons, we compare the following cases. Figure 3 depicts the intensity of two soliton as a function according to the propagation direction z for the different pulse compression factor z_0. As shown in Fig. 3, we note that as $|z_0|$ decreases, the amplitude of the pulse increases with compression in its width, and the interaction between two neighboring solitons decreases, hence the compression of the pulse can be achieved.

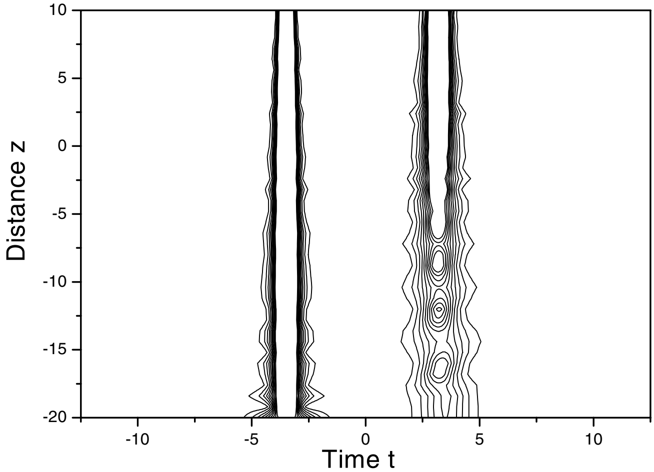


Fig. 2. Interaction of two unequal amplitude pulses with initial pulse separation equal to 7, the parameters are as follows: $A = 1.5$ and the compression factor $z_0 = -22.6$.

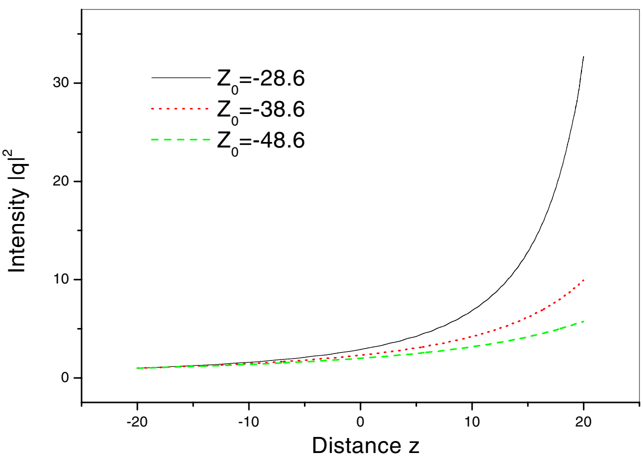


Fig. 3. Variation of the pulse intensity with respect to propagation distance for different compression factors z_0.

4. Conclusions

In this paper, we construct the Lax pair for a soliton transmission system in nonuniform optical fibers and give N-soliton solution using the simple straightforward Darboux transformation. And the explicit one-soliton and two-soliton solutions are presented. Further, we discuss in detail the interaction scenario between two neighboring solitons as well as the effect of the inhomogeneities of the fiber (z_0) on the interaction between two neighboring solitons. Pulse compression and soliton switching have been observed by choosing suitable parameters, which is helpful for the next generation high-bit optical fiber communication systems.

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References

1. A. Hasegawa and F. Tappert, *Appl. Phys. Lett.* **23**, 142 (1973).
2. A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford University Press, New York, 1995).
3. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, 1989).
4. F. Abdullaev, S. Darmanyan and P. Khabibullaev, *Opticals Solitons* (Springer, Berlin, 1991).
5. H. H. Kuehl, *J. Opt. Soc. Am.* **B5**, 709 (1988).
6. D. Anderson and M. Lisak, *Phys. Rev.* **A35**, 184 (1987).
7. S. P. Burtsev, A. V. Mikhailov and V. E. Zakharov, *Theor. Math. Phys.* **70**, 227 (1987).
8. K. Nakkeeran, *Nonlinearity* **15**, 1747 (2002).
9. Z. Y. Xu, L. Li, Z. H. Li and G. S. Zhou, *Opt. Commun.* **210**, 375 (2002).
10. Z. Y. Xu, L. Li, Z. H. Li and G. S. Zhou, *Phys. Rev.* **E67**, 026603 (2003).
11. V. B. Matveev and M. A. Salli, *Darboux Transformations and Solitons*, Springer Series in Nonlinear Dynamics (Springer-Verlag, Berlin, 1991).