

Self-tuning mechanisms of nonlinear split-ring resonators

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We study both theoretically and experimentally the dynamic tunability of the magnetic resonance of a single nonlinear split-ring resonator with varactor diode at microwave frequencies. We demonstrate different tuning regimes with and without an inductive coil in parallel with the varactor. We show that the coil changes the sign of the nonlinearity and eliminates the memory effect caused by charge accumulation across the varactor. In addition, at higher powers the nonlinear response of the split-ring resonator becomes multivalued, paving a way for creating bistable tunable metamaterials. © 2007 American Institute of Physics. [DOI: 10.1063/1.2794733]

Specially engineered microstructured metamaterials demonstrating many intriguing properties for the propagation of electromagnetic waves such as negative refraction have been studied extensively in recent years (see, e.g., the review paper¹ and references therein). Typically, metamaterials are periodic structures obtained by stacking many scattering objects with size much smaller than the wavelength of electromagnetic radiation and often such composite materials can be described in terms of macroscopic quantities—the electric permittivity ϵ and the magnetic permeability μ . By carefully designing the individual unit cell of metamaterials, one may construct composites with effective properties not occurring in nature.

Split-ring resonators (SRRs) are the key building blocks for metamaterials, particularly those having negative refractive index.² Recent studies have demonstrated how to dynamically tune or modulate the electromagnetic properties of metamaterials,^{3–7} and the fabrication of nonlinear SRRs has been demonstrated by placing a varactor diode⁸ or a photosensitive semiconductor⁹ within the gap of the resonator. The diode allows the SRR to be tuned by an applied dc voltage or by a high power signal. In this letter, we report on experimental and theoretical studies which explain the self-tuning mechanisms of SRRs and experimentally demonstrate the multivalued behavior predicted earlier.³

We study theoretically the nonlinear self-tuning of the SRR shown schematically in Fig. 1. We distinguish between the cases with and without an inductive coil placed in parallel with the varactor. With the coil, the configuration is essentially the same as that reported in Ref. 8. The constant voltage across the varactor due to signal rectification is eliminated by the coil, which has large inductive reactance causing its influence on the circuit to be negligible at the frequency of the microwave signal.

As shown in earlier studies,⁸ an increase in applied power causes an increase in the resonant frequency of the split-ring resonator. At the same time, other groups have reported a decrease of the resonant frequency with an increase of applied power.¹⁰ This difference can be explained by the

different configurations of the varactor diode. Indeed, when the coil is absent, the constant biasing shifts the operating point of the varactor to the region of the I - V curve with low effective conductance and low capacitance. Accordingly, the SRR resonant frequency grows. With the inductive coil, the varactor operates around the zero voltage point and, as we show below, this reverses the sign of the nonlinearity.

The equivalent circuit of the split-ring resonator is shown in Fig. 1. The ring is represented by an RLC circuit, and the diode is represented by a parallel nonlinear conductance $G(V)$ and nonlinear capacitance $C(V)$. A more accurate model of the varactor would include some resistance and inductance in series. However, these can be combined with the inductance and capacitance of the SRR, and the model does not change qualitatively.

The small-signal nonlinear capacitance and conductance are defined from the equivalent currents,

$$G(V) = dI_G(V)/dV,$$

$$C(V) = dQ_C(V)/dV, \quad (1)$$

where the charge accumulated by the varactor capacitance is $Q_C = \int_0^t I_C(t') dt' + Q_C(0)$.

Assuming that the minority carrier diffusion current and depletion capacitance are the dominant mechanisms,

$$I_G(V) = I_s [\exp(V/\eta V_T) - 1],$$

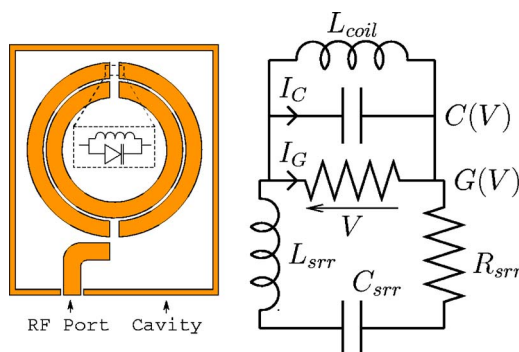


FIG. 1. (Color online) (Left) Schematic view of a varactor-loaded split-ring resonator system in a cavity. (Right) Equivalent circuit.

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$$C(V) = C_0(1 - V/V_f)^{-M}, \quad (2)$$

where M , I_s , η , C_0 , and V_f are available from the diode specification, and V_T is the thermal voltage.

The power-dependent frequency shift of the resonator is not described by the small-signal capacitance C , so an effective diode capacitance C_{eff} is introduced, which includes the influence of the finite signal amplitude. Assuming that the rf response of the varactor is dominated by its capacitance, the equivalent impedance within the loop of the SRR will be

$$Z(\omega) = -\frac{iL_{\text{SRR}}}{\omega} \left(\omega^2 - \omega_0^2 - \frac{1}{L_{\text{SRR}}C_{\text{eff}}} + \frac{i\omega R}{L_{\text{SRR}}} \right). \quad (3)$$

The varactor alters the resonant frequency ω_r as

$$\omega_r = \omega_0(1 + C_{\text{SRR}}/C_{\text{eff}})^{1/2}, \quad (4)$$

where $\omega_0 = (L_{\text{SRR}}C_{\text{SRR}})^{-1/2}$.

A full analysis of the nonlinear SRR must include all harmonics of the currents and voltages. However, to demonstrate the self-tuning behavior, we explore here the simplest case of the varactor subjected to a voltage with a single dominant harmonic,

$$V(t) = V_0 + V_1 \cos \omega t. \quad (5)$$

This approximation is expected to remain valid in an intermediate power regime, where higher harmonics of voltage are present, but are not strong enough to induce a change in the diode response. It is also noted that the capacitive susceptance of the diode increases linearly with frequency, thus it tends to suppress voltages at higher harmonics by short circuiting them.

We substitute Eq. (5) into Eq. (2), and note that in the absence of the coil, the constant component of the current must vanish. This gives V_0 as

$$V_0 = -V_T \ln[I_0(V_1/V_T)], \quad (6)$$

where I_0 is a modified Bessel function of the first kind. For $V_1 \gg V_T$ this relationship can be approximated by $V_0 = V_f - V_1$, due to the diode clamping the maximum positive voltage at the forward conduction voltage V_f .

We then take the third-order expansion of $Q_C(V)$ about V_0 and find its Fourier component of frequency ω . This yields the effective capacitance of the varactor without the coil,

$$C_{\text{eff}} = C(V_0) + \frac{3}{8}C''(V_0)V_1^2. \quad (7)$$

Evaluating this for the specified model parameters in accordance with Eqs. (2) and (6) we find that the first term in Eq. (7) dominates, and the effective capacitance is reduced.

In the presence of the coil $V_0=0$, and the second term alone describes the self-tuning. As follows from Eq. (2), $C''(0) = C_0M(M+1)V_f^{-2}$. Since the parameters C_0 , M , and V_f are positive, this results in an increase of C_{eff} .

Next, we study the self-tuning experimentally at microwave frequencies. The SRR was fabricated with inner radius of 2.56 mm, track width of 1.44 mm, gap between rings and in slits with width of 0.32 mm, and an additional gap for the varactor of 0.4 mm. In order to experimentally find the value of C_{SRR} , we placed lumped linear capacitors in the split of the resonator and measured the corresponding change of the resonant frequency. By fitting the results to Eq. (4), it was

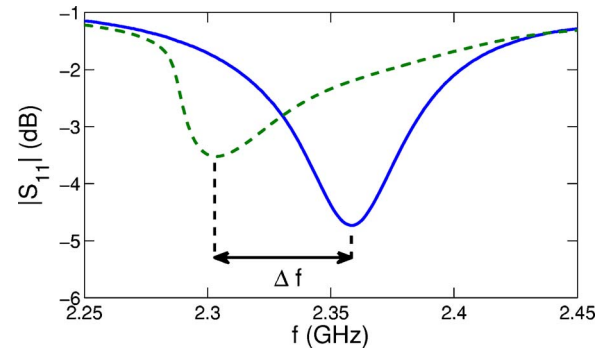


FIG. 2. (Color online) Reflection S_{11} for nonlinear split-ring resonator with a coil for the input power 0 dBm (solid line) and 15 dBm (dashed line).

found that $C_{\text{SRR}} \approx 0.35$ pF. However, the high uncertainty in small capacitance values causes significant error. Empirically, we have found the value 0.24 pF to give better agreement with nonlinear measurements, so this value has been used. The nonlinearity was introduced into the split-ring resonator by inserting an SMV1405 varactor.

In order to suppress radiation losses, the SRR was placed inside a conductive cavity with open ends, as shown in Fig. 1. The cavity cross section is small so that the SRR resonant frequency is well below the resonant frequency of the cavity. The reduction in radiation from the SRR greatly improved its quality factor and enhanced the nonlinear effects. The effect of the cavity on the SRR reactance can be understood in the quasistatic approximation as causing image currents to flow in the SRR plane. Inductive interaction with those virtual SRRs provides a small contribution to the SRR inductance, which slightly changes the resonant frequency.

Measurement of the nonlinear SRR was performed via the the rf port shown in Fig. 1. The reflection coefficient was measured for different values of the input power, and a typical example is shown in Fig. 2.

We have found that the diode discharges very slowly without the coil. Thus it was necessary to gradually scan through the frequency range to ensure that the measurements at each frequency were not affected by the rectification occurring at another. The resonant frequency was determined from the minimum of the reflection coefficient, and its shift is presented in Fig. 3 for the cases with and without a coil.

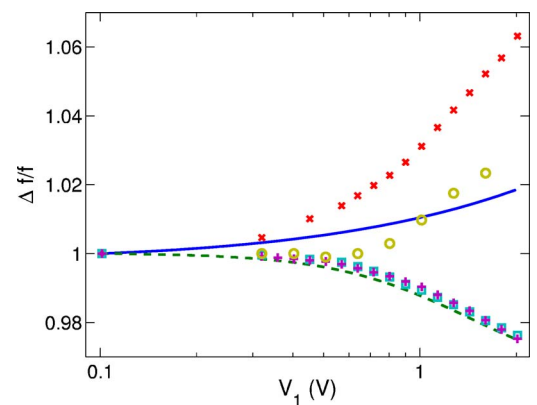


FIG. 3. (Color online) Shift of the resonant frequency of the split-ring resonator vs diode rf voltage found from Eq. (4) without (solid line) and with (dashed line) coil, calculated numerically without (○) and with (×) coil, and measured without (×) and with (□) coil.

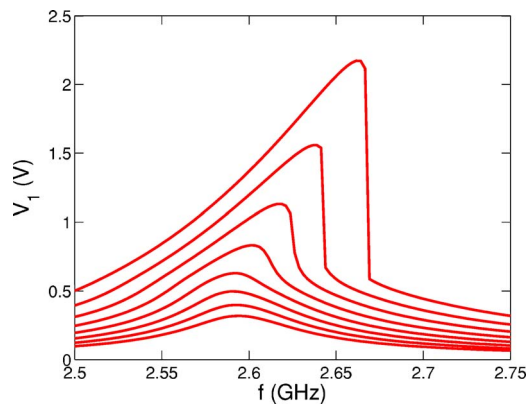


FIG. 4. (Color online) rf voltage across diode for rf power between 0 and 14 dBm without coil.

The theoretical results obtained from Eq. (4) with fitted values of C_{SRR} are also plotted.

In order to compare the theoretical results with the experimental data, it was necessary to find the relationship between V_1 and the power applied to the rf port P_{rf} . The method of moments was used to find the frequency response of the split-ring resonator. A harmonic balance simulation was performed in order to obtain the steady-state reflection from the complete nonlinear circuit. It was found that $V_1(f_r) \approx 0.32\sqrt{P_{\text{rf}}}$, and the numerically calculated frequency shift was also added to Fig. 3.

For the case with the coil, good agreement is found between numerical, analytical, and measured results. For the case without the coil the agreement is poor, due to a number of reasons. Firstly, the analytical model of the self-biasing voltage is based on the simplest possible I - V relationship for a diode, since the SPICE parameters for other conduction mechanisms are not available for the SMV1405. Secondly, in order to improve convergence the SPICE model of a diode includes nonphysical conductance terms which greatly overestimate the reverse bias current. Finally, the nonlinear resonance without the coil becomes multivalued at high input power, thus causing the harmonic balance simulation to become unreliable. Figure 4 illustrates this phenomena, showing the sudden discontinuity in V_1 as a function of frequency. In the measured reflection coefficient in Fig. 5 the multivalued behavior can be clearly seen.

In conclusion, we have demonstrated control of the sign of the nonlinear response of an SRR at microwave frequencies by choosing a setup with or without an additional coil.

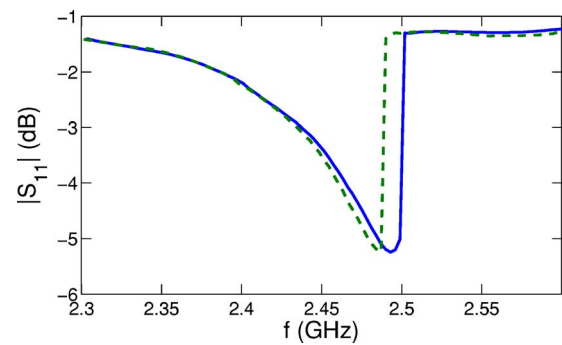


FIG. 5. (Color online) S_{11} showing multivalued behavior for 15 dBm input power without coil: (solid line) forward sweep and (dashed line) reverse sweep.

The configuration without the coil shows stronger nonlinearity, but it has a pronounced memory effect, which slows down the response to changes in the input power. Additionally, the resonator without the coil was found to be highly sensitive to interference from electrical wiring within the building. This problem was largely eliminated once the resonator was placed inside the cavity. We believe that our results pave a way for creating nonlinear active metamaterials composed of tunable split-ring resonators.

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