Estimation of stochastic processes from financial data

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Our contribution concerns with the common feature that the statistics of returns for financial assets are dependent on the time scale [1][2]. This poses great challenges to risk management and other applications. A stochastic analysis of financial data is presented. The considered data sets consist of high frequency data containing some 10^6 data-points for the years 1993-2003. Evidence is given that the log return $R(\tau)$ upon different time scales τ can be described as a Markov process evolving in τ. Thus the τ-dependence of the probability density functions (pdf) $p(R, \tau)$ on the time scale τ can be described by a Fokker-Planck equation, a generalized diffusion equation for $p(R, \tau)$ [3]. With such a stochastic approach the general joint pdf for arbitrarily many different time scales τ_i can be described providing a non-equilibrium thermodynamical description of the complexity of the data. We demonstrate by a new improved method how this Fokker-Planck equation can be estimated directly from the data in a parameter free way. It is shown that the solutions of the resulting Fokker-Planck equation describe the empirical pdfs correctly, including the pronounced fat tails, which are of particular interest for risk management. Furthermore in the range of small τ where it has been shown, that the process is no longer a Markov process, a method is introduced in order to analyse the shape evolution of the non Gaussian pdfs with time scale parameter τ. A detailed analysis reveals a special small scale regime, which can be clearly distinguished from the behaviour on larger scales and which seems to be universal for stocks.

[1] RN Mantegna and HE Stanley, Nature 376, 6535 (1995)
[2] S Ghashghaie, W Breymann, J Peinke, et. al., Nature 381, 6585 (1996)
[3] C Renner, J Peinke, R Friedrich, et. al., PRL 89, 12 (2002)