

A Free Boundary Problem Related to Environmental Management System

Robert Elliott ⁽¹⁾, Alexei Filinkov ⁽²⁾ and Robin Nicholson ⁽³⁾

¹ Haskayne School of Business, University of Calgary, Calgary, Alberta, Canada T2N 1N4

² Land Operations Division, Defence Science and Technology Organisation, Edinburgh SA 5111, Australia; School of Mathematical Sciences, University of Adelaide, SA 5005, Australia

³ Land Operations Division, Defence Science and Technology Organisation, Edinburgh SA 5111, Australia

Corresponding author e-mail: alexei.filinkov@adelaide.edu.au

In paper [1] Pindyck considers the optimal time to reduce the rate of pollution in an environmental model. He considers two processes, one of which (S) represents the level of pollution or degradation in some environmental area. The second (θ) models the social, political or financial cost of the pollution. Pindyck considers a single control parameter which models reducing the rate of pollution. It is assumed that this reduction can be made only once and it cannot be reversed. This is justified by observing that such irreversibility is often inherent in environmental policy: once decisions have been made, they are not usually overturned.

Pindyck initially supposes that θ switches only at a known time T to one of two values, $\underline{\theta}$ or $\bar{\theta}$, where $\underline{\theta} \leq \theta \leq \bar{\theta}$. More interestingly later in his paper he assumes θ is described by a log-normal process. However, only in a special case is an explicit solution is obtained.

In this paper we suppose the level of pollution is described by a log-normal process whose dynamics involve a control parameter u . We briefly discuss the relative advantages of switching the control when θ only jumps to $\bar{\theta}$ or $\underline{\theta}$ at time T . Proceeding to the case of log-normal dynamics for θ we can give an explicit solution of a free boundary problem which provides the optimal time to reduce u_0 to u_1 .

[1] R.S. Pindyck, *Optimal timing problems in invironmental economics*. J. Econ. Dynamics and Control, 2002 (26), 1677 - 1697.