

13th INTERNATIONAL STELLARATOR WORKSHOP

GLOBAL BALLOONING MODES IN LOW-SHEAR STELLARATORS

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The calculation of the finite-mode-number spectrum of linear modes in stellarators is a challenging problem due to the coupling of both toroidal and poloidal Fourier harmonics. In a spectral approach, this leads to very large matrices. An alternative approach to this problem for low-frequency, short-wavelength waves and instabilities is to use the WKB-ballooning formalism [1] to construct local approximate solutions, using ray tracing and a “semiclassical quantization” formula to give the global eigenfrequencies/growth rates.

If the rays lie on topological tori, the quantization formulae are in terms of loop integrals around the topologically distinct circuits of the torus [Einstein–Brillouin–Keller (EBK) quantization]. In the opposite limit of strongly chaotic rays, the eigenvalue spectrum is effectively random, and must be described statistically (“quantum chaos”).

For axisymmetric tokamaks, and for weakly localized (e.g. interchange) modes in stellarators, the field line label,  $\alpha$ , is an ignorable coordinate and EBK quantization is possible, providing good estimates of the spectrum [2]. However for modes in stellarators for which the local ideal ballooning growth rate depends strongly on  $\alpha$ , EBK quantization breaks down due to the phase-space rays escaping to infinity in perpendicular wavelength [1]. This runaway effect is prevented by including finite-Larmor-radius (FLR) effects or by an ideal-MHD regularization procedure simulating spectral truncation, but then the ray orbits are found to be chaotic [4].

The local ballooning growth rate has been found to depend strongly on  $\alpha$  in H-1NF Helic [3] and NCSX studies [5], while the ballooning angle parameter,  $\theta_k$ , dependence is weaker due to low global magnetic shear. This paper presents studies of the resulting ray dynamics and discusses the implications for the ballooning mode spectrum in H-1NF and NCSX.

[1] R.L. Dewar and A.H. Glasser, *Ballooning mode spectrum in general toroidal systems*, Phys. Fluids **26**, 3038 (1983).

[2] W.A. Cooper, D.B. Singleton, and R.L. Dewar, *Spectrum of ballooning instabilities in a stellarator*, Phys. Plasmas **3**, 275 (1996); Erratum Phys. Plasmas **3**, 3520 (1996).


[3] P. Cuthbert and R.L. Dewar, *Anderson-localized ballooning modes in general toroidal plasmas*, Phys Plasmas **7**, 2302 (2000).

[4] R.L. Dewar, P. Cuthbert and R. Ball, *Strong “Quantum” Chaos in the Global Ballooning Mode Spectrum of Three-Dimensional Plasmas*, Phys. Rev. Letters, **86**, 2321 (2001).


[5] M.H. Redi *et al.*, *Ballooning stability of the compact quasiaxially symmetric stellarator*, this conference.


Conference topic: (3) MHD Equilibrium and Stability

## Global Ballooning Modes in Low-Shear Stellarators



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## Plan of talk

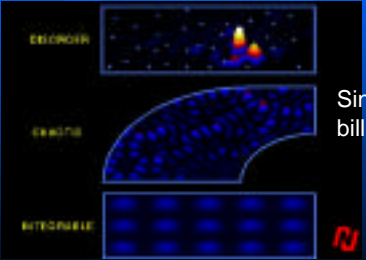
- Regular and chaotic semiclassical quantization
- Ballooning parameter spaced studies on H-1 and NCSX
- Regularization of ideal MHD WKB-ballooning theory
- Chaotic ray paths for toroidally localized modes and Weyl estimate for  $\min. n_{\max}$ .

## Calculation of global eigenvalues

- Galerkin method:
  - Expand in basis set
  - Diagonalize matrix
  - CAS3D and TERPSICHORE
- Semiclassical (WKB) method:
  - Find local dispersion relation
  - Calculate ray dynamics
  - Quantize by calculating phase integrals *if possible* (rays bounded and not chaotic)

## “Quantum” chaos with microwaves (Sridhar\*)

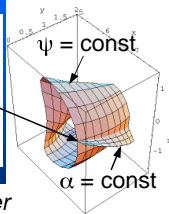
Sinai billiard



\*<http://sagar.physics.neu.edu/index.html>

## Clebsch representation: $\mathbf{B} = \alpha \times \psi$

**B**-lines can be defined by the *intersection* of surfaces of constant  $\psi$  and  $\alpha$ .  
By choosing appropriate angles, we make  $\alpha = \zeta - q\theta$ .



Here  $q(s)$  is the *rotation number* (inverse rotational transform) of the field lines on surface  $s$ :

$$q = \lim_{l \rightarrow \infty} \frac{\zeta}{\theta}$$

## WKB Ballooning theory

- Straight-field-line coordinates  

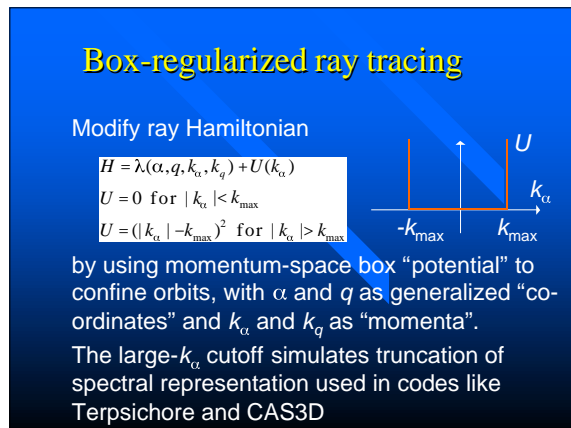
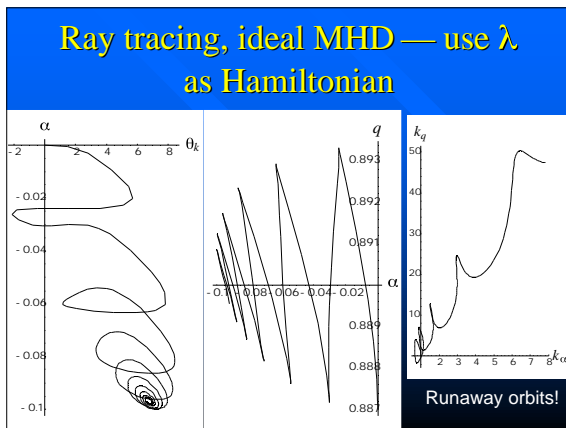
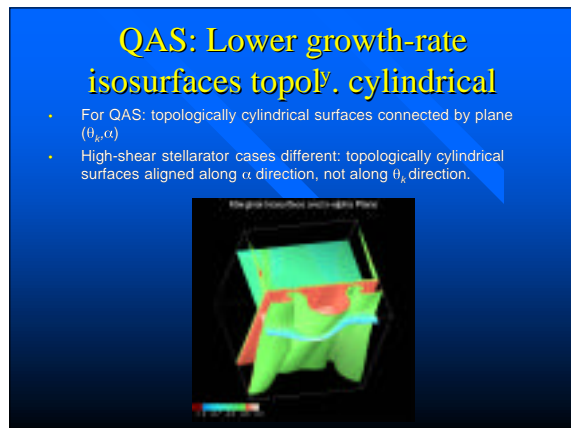
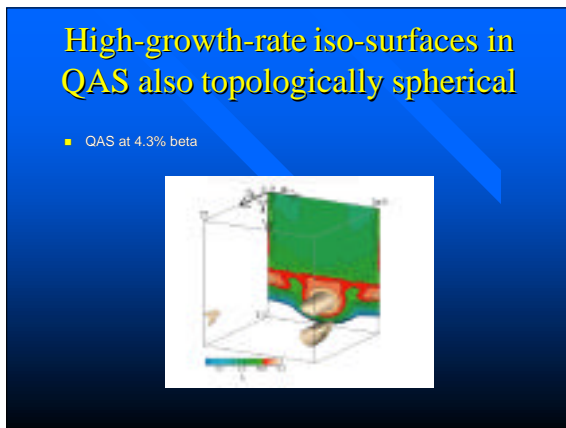
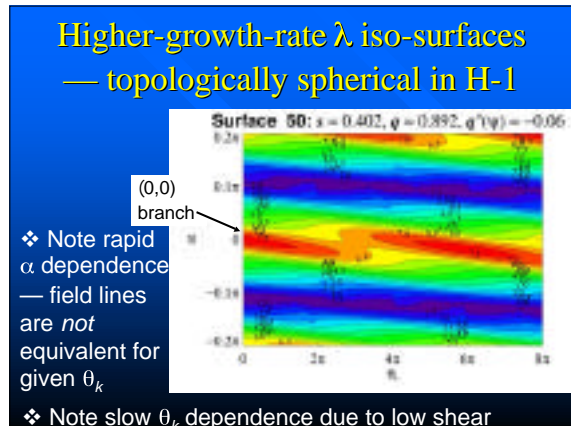
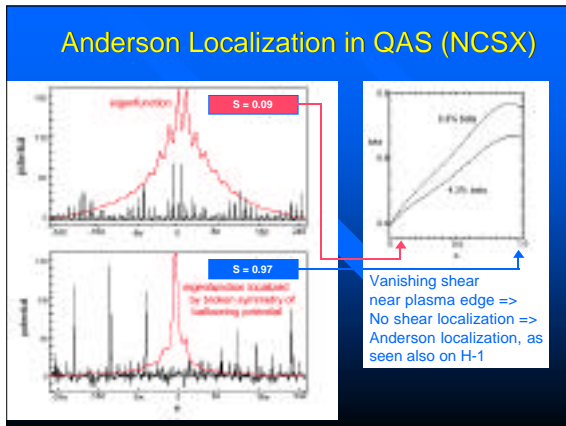
$$\mathbf{B} = \alpha \times \psi = \alpha \times q/q(\psi)$$
- Eikonal Ansatz for flute-like modes  

$$\varphi = \hat{\varphi} \exp[iS(\alpha, q)/\epsilon - i\omega t]$$
- Wave vector  

$$\mathbf{k} = k_\alpha \alpha + k_q q + k_\alpha [\zeta - q\theta - (\theta - \theta_k) q]$$
- Ballooning eigenvalue equation  

$$\frac{d}{d\theta} \mathcal{A} \frac{d}{d\theta} - \mathcal{K} + \lambda \mathcal{N} \varphi = 0 \quad \text{where } \lambda = \rho \omega^2$$

Leads to 2D dispersion relation



## Poincaré plots: $k_{\max} = 50, \lambda = -6$



Orbits fill “energetically” accessible phase space, (except they do not cross  $k = 0$ ) — *strong chaos*



## Weyl formula

For a 2D chaotic system, ignoring fine details predicted by Gutzwiller trace formula, the number of states with growth rate greater than  $\gamma^2$  is  $(1/2\pi)^2$  X phase space volume within the region

$$\frac{|k_\alpha| k_{\max}}{\lambda(s, \alpha, \theta_k)} \gamma^2$$

For  $\lambda = -6$  contour this gives minimum

$$k_{\max} = n_{\max} = 60 \text{ to fit one mode H-1NF}$$

FLR-stabilized wrt ideal toroidally localized ballooning?

## Conclusions

- Most unstable ideal-MHD ballooning modes in H-1 and NCSX edge region are toroidally localized due to Anderson localization
- “Semiclassical quantization” breaks down due to unbounded ray paths
- Ideal-MHD can be regularized by cutoff at high- $k_\alpha$  reflecting boundary cond.
- Quantum chaotic global spectrum coupling of all  $n$ 's. High bandwidth required to resolve with spectral codes.